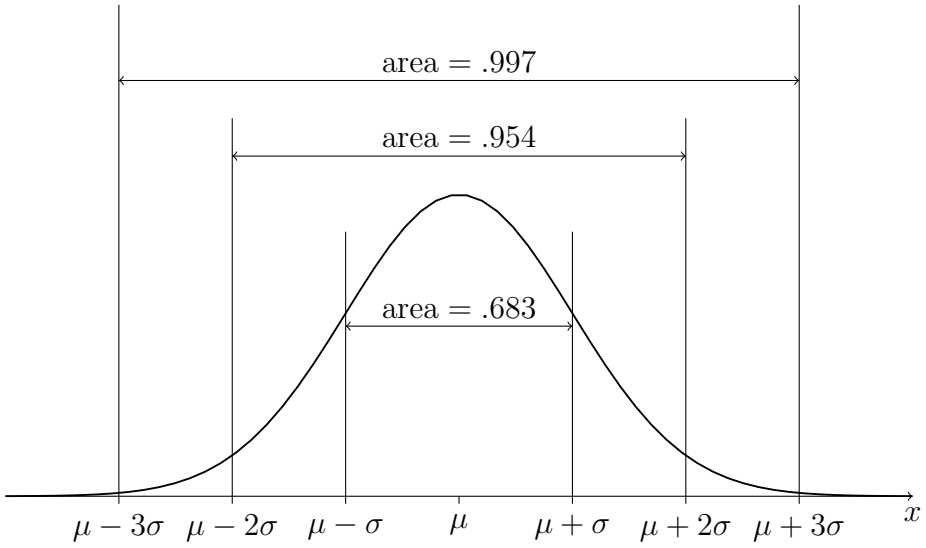
STAT 515 sp 2024 Exam I

Karl Gregory

- Do not open this exam until told to do so.
- You may have one handwritten sheet of notes out during the exam.
- You have 75 minutes to work on this exam.
- You may NOT use any kind of calculator.
- If you are unsure of what a question is asking for, do not hesitate to ask me for clarification.
- *Good luck, and may the odds be ever in your favor!*

$X \sim$		$\mathcal{X}$	$\mathbb{E}X$	$\text{Var}(X)$
$\text{Binomial}(n,p)$	$P(X=x)=\binom{n}{x}p^x(1-p)^{n-x}$	$x=0,1,\ldots,n$	np	$np(1-p)$



1. Among the patrons of a library, 80% are at least thirty years old. Those at least thirty years old borrow a hard-copy book 70% of the time and an ebook 30% of the time. Those younger than thirty borrow a hard-copy book 40% of the time an ebook 60% of the time.

   (a) Give the probability that the next book borrowed by a randomly selected patron is a hard-copy book.

   (b) If a randomly selected patron borrows a hard-copy book, give the probability that the patron was thirty years old or older.

2. A grower of Pink Lady apples brings to market apples weighing, on average, 100 grams. Suppose the standard deviation of the apple weights is 5 grams and that the weights have a Normal distribution.

   (a) What proportion of the apples have weights between 90 and 110 grams?

   (b) With what probability would a randomly selected apple weigh more than 105 grams?

   (c) Give an interval such that 99.7% of apples from this grower would have a weight in the interval.

3. Consider the phrase *all mimsy were the borogoves*.
   (a) How many sequences of words can you make by rearranging the words in the phrase?

   (b) In a random rearrangement, with what probability will *borogoves* be one of the first two words?

   (c) How many unique sequences of 5 letters can you make by rearranging the letters in *mimsy*?

   (d) In how many ways can you choose two words in the phrase to cross out?

   (e) In how many ways can you choose three words in the phrase to cross out?

4. For three applicants to a graduate program, let A_1, A_2, and A_3 be the events that the applicants are accepted. Express the following events using elementary set operations on A_1, A_2, and A_3.
   (a) At least one of the applicants is accepted.

   (b) None of the applicants is accepted.

   (c) Exactly two of the applicants are accepted.

5. Suppose a breed of dog has litter sizes 1,2,...,7 with the probabilities given in the table:

litter size	1	2	3	4	5	6	7
probability	0.1	0.2	0.3	0.2	0.1	0.05	0.05

(a) Give the probability of a litter size of at least 2 puppies.

(b) Give a table showing the cumulative probabilities for the litter sizes, that is $P(X \leq x)$, for each $x = 1, 2, \ldots, 7$, where X is the litter size.

(c) Give the expected value of the litter size.

6. Suppose a six-sided die is rolled five times. Let X be the number of ⚅'s rolled.

(a) What is the name of the probability distribution of X?

(b) Give an expression (you do not need to evaluate it) for $P(X = 3)$.

(c) Give the probability that you will roll all ⚅'s.

(d) Give the expected value of X.