

STAT 516 Lec 10

Split-plot design

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Alfalfa data from Dr. Longnecker's notes

- ▶ Six fields; three plots in each field; four sub-plots in each plot.
- ▶ Each plot randomly assigned a type of alfalfa.
- ▶ Each sub-plot randomly assigned a cutting date.
- ▶ Response for each subplot is yield in tons/acre in the following year.

Variety	Date	Blocks						TrT Mean \bar{Y}_{ij}
		1	2	3	4	5	6	
Ladak	None	2.17	1.88	1.62	2.34	1.58	1.66	1.8750
	S1	1.58	1.26	1.22	1.59	1.25	0.94	1.3067
	S20	2.29	1.60	1.67	1.91	1.39	1.12	1.6633
	O7	2.23	2.01	1.82	2.10	1.66	1.10	1.8200
Cossack	None	2.33	2.01	1.70	1.78	1.42	1.35	1.7650
	S1	1.38	1.30	1.85	1.09	1.13	1.06	1.3017
	S20	1.86	1.70	1.81	1.54	1.67	0.88	1.5767
	O7	2.27	1.81	2.01	1.40	1.31	1.06	1.6433
Ranger	None	1.75	1.95	2.13	1.78	1.31	1.30	1.7033
	S1	1.52	1.47	1.80	1.37	1.01	1.31	1.4133
	S20	1.55	1.61	1.82	1.56	1.23	1.13	1.4833
	O7	1.56	1.72	1.99	1.55	1.51	1.33	1.6100

```
alfalfa <- data.frame(yield = c(2.17,1.88,1.62,2.34,1.58,1.66,  
                               1.56,1.26,1.22,1.59,1.25,0.94,  
                               2.29,1.60,1.67,1.91,1.39,1.12,  
                               2.23,2.01,1.82,2.10,1.66,1.10,  
                               2.33,2.01,1.70,1.78,1.42,1.35,  
                               1.38,1.30,1.85,1.09,1.13,1.06,  
                               1.86,1.70,1.81,1.54,1.67,0.88,  
                               2.27,1.81,2.01,1.40,1.31,1.06,  
                               1.75,1.95,2.13,1.78,1.31,1.30,  
                               1.52,1.47,1.80,1.37,1.01,1.31,  
                               1.55,1.61,1.82,1.56,1.23,1.13,  
                               1.56,1.72,1.99,1.55,1.51,1.33),  
                     variety = as.factor(c(rep("ladak",24),  
                                           rep("cossack",24),  
                                           rep("ranger",24))),  
                     date = as.factor(rep(c(rep("4none",6),  
                                           rep("1sep01",6),  
                                           rep("2sep20",6),  
                                           rep("3oct07",6)),  
                                         3)),  
                     field = as.factor(rep(1:6,12)))
```

```
head(alfalfa,n = 26)
```

	yield	variety	date	field
1	2.17	ladak	4none	1
2	1.88	ladak	4none	2
3	1.62	ladak	4none	3
4	2.34	ladak	4none	4
5	1.58	ladak	4none	5
6	1.66	ladak	4none	6
7	1.56	ladak	1sep01	1
8	1.26	ladak	1sep01	2
9	1.22	ladak	1sep01	3
10	1.59	ladak	1sep01	4
11	1.25	ladak	1sep01	5
12	0.94	ladak	1sep01	6
13	2.29	ladak	2sep20	1
14	1.60	ladak	2sep20	2
15	1.67	ladak	2sep20	3
16	1.91	ladak	2sep20	4
17	1.39	ladak	2sep20	5
18	1.12	ladak	2sep20	6
19	2.23	ladak	3oct07	1
20	2.01	ladak	3oct07	2
21	1.82	ladak	3oct07	3
22	2.10	ladak	3oct07	4
23	1.66	ladak	3oct07	5
24	1.10	ladak	3oct07	6
25	2.33	cossack	4none	1
26	2.01	cossack	4none	2

Randomized complete block split plot design

- ▶ Each EU randomly assigned to one level of whole-plot factor.
- ▶ Each EU receives all levels of split-plot factor in random order.
- ▶ Groups of EUs over which this is replicated are treated as blocks.

Treatment effects model for RCB split-plot design

Assume

$$Y_{ijk} = \mu + \tau_i + \gamma_j + (\tau\gamma)_{ij} + C_k + (\tau C)_{ik} + \varepsilon_{ijk},$$

for $i = 1, \dots, a$, $j = 1, \dots, b$ and $k = 1, \dots, c$, where

- ▶ Y_{ijk} is the response of sub-plot j from whole plot i in block k .
- ▶ the τ_i are treatment effects for the whole-plot factor.
- ▶ the γ_j are treatment effects for the split-plot factor.
- ▶ the $(\tau\gamma)_{ij}$ are interaction effects between the factors.
- ▶ the C_k are independent $\text{Normal}(0, \sigma_C^2)$ block effects.
- ▶ the $(\tau C)_{ik}$ are independent $\text{Normal}(0, \sigma_{AC}^2)$ whole-plot effects.
- ▶ the ε_{ijk} are independent $\text{Normal}(0, \sigma_\varepsilon^2)$ error terms.

Define the cell means as

$$\mu_{ij} = \mu + \tau_i + \gamma_j + (\tau\gamma)_{ij}, \quad i = 1, \dots, a, \quad j = 1, \dots, b.$$

Sums of squares for the RCB split-plot design

SS	Symbol	Formula
Total	SS_{Tot}	$\sum_{i=1}^a \sum_{j=1}^b \sum_{k=1}^c (Y_{ijk} - \bar{Y}_{...})^2$
A	SS_A	$bc \sum_{i=1}^a (\bar{Y}_{i..} - \bar{Y}_{...})^2$
C	SS_C	$ab \sum_{k=1}^c (\bar{Y}_{..k} - \bar{Y}_{...})^2$
AC	SS_{AC}	$b \sum_{i=1}^a \sum_{k=1}^c (\bar{Y}_{i.k} - (\bar{Y}_{i..} + \bar{Y}_{..k} - \bar{Y}_{...}))^2$
B	SS_B	$ac \sum_{j=1}^b (\bar{Y}_{.j.} - \bar{Y}_{...})^2$
AB	SS_{AB}	$c \sum_{i=1}^a \sum_{j=1}^b (Y_{ij.} - (\bar{Y}_{i..} + \bar{Y}_{.j.} - \bar{Y}_{...}))^2$
Error	SS_{Error}	$SS_{\text{Tot}} - (SS_A + SS_B + SS_{AB} + SS_C + SS_{AC})$

► We have $SS_{\text{Tot}} = SS_A + SS_B + SS_{AB} + SS_C + SS_{AC}$.

ANOVA table for RCB split-plot design

Source	Df	SS	MS	F value
A	$a - 1$	SS_A	MS_A	$F_A = MS_A / MS_{AC}$
C	$c - 1$	SS_C	MS_C	$F_C = MS_C / MS_{AC}$
AC	$(a - 1)(c - 1)$	SS_{AC}	MS_{AC}	$F_{AC} = MS_{AC} / MS_{Error}$
B	$b - 1$	SS_B	MS_B	$F_B = MS_B / MS_{Error}$
AB	$(a - 1)(b - 1)$	SS_{AB}	MS_{AB}	$F_{AB} = MS_{AB} / MS_{Error}$
Error	$a(b - 1)(c - 1)$	SS_{Error}	MS_{Error}	
Total	$abc - 1$	SS_{Tot}		

1. Reject $H_0: \bar{\mu}_{1.} = \dots = \bar{\mu}_{a.}$ if $F_A > F_{a-1, (a-1)(c-1), \alpha}$.
2. Reject $H_0: \sigma_C^2 = 0$ if $F_C > F_{c-1, (a-1)(c-1), \alpha}$.
3. Reject $H_0: \sigma_{AC}^2 = 0$ if $F_{AC} > F_{(a-1)(c-1), a(b-1)(c-1), \alpha}$.
4. Reject $H_0: \bar{\mu}_{.1} = \dots = \bar{\mu}_{.b}$ if $F_B > F_{b-1, a(b-1)(c-1), \alpha}$.
5. R. $H_0: \mu_{ij} = \bar{\mu}_{i.} + \bar{\mu}_{.j} - \bar{\mu}_{..} \quad \forall ij$ if $F_{AB} > F_{(a-1)(b-1), a(b-1)(c-1), \alpha}$.

Expected mean squares in RCB split plot design

Source	Df	Expected mean square
A	$a - 1$	$bc\theta_A^2 + b\sigma_{AC}^2 + \sigma_\varepsilon^2$
C	$c - 1$	$ab\sigma_C^2 + b\sigma_{AC}^2 + \sigma_\varepsilon^2$
AC	$(a - 1)(c - 1)$	$b\sigma_{AC}^2 + \sigma_\varepsilon^2$
B	$b - 1$	$ac\theta_B^2 + \sigma_\varepsilon^2$
AB	$(a - 1)(b - 1)$	$c\theta_{AB}^2 + \sigma_\varepsilon^2$
Error	$a(b - 1)(c - 1)$	σ_ε^2

In the above

$$\blacktriangleright \theta_A^2 = (a - 1)^{-1} \sum_{i=1}^a (\bar{\mu}_{i.} - \bar{\mu}_{..})^2$$

$$\blacktriangleright \theta_B^2 = (b - 1)^{-1} \sum_{j=1}^b (\bar{\mu}_{.j} - \bar{\mu}_{..})^2$$

$$\blacktriangleright \theta_{AB}^2 = [(a - 1)(b - 1)]^{-1} \sum_{i=1}^a \sum_{j=1}^b (\mu_{ij} - (\bar{\mu}_{i.} + \bar{\mu}_{.j} - \bar{\mu}_{..}))^2$$

Alfalfa data (cont)

`anova()` on `lm()` output gives wrong p-vals for variety and field.

```
lm_out <- lm(yield ~ variety + date + variety:date + field + field:variety,  
            data = alfalfa)  
anova(lm_out)
```

Analysis of Variance Table

Response: yield

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
variety	2	0.1753	0.08763	3.1198	0.0538447 .
date	3	1.9727	0.65758	23.4123	2.789e-09 ***
field	5	4.1388	0.82775	29.4710	3.798e-13 ***
variety:date	6	0.2147	0.03579	1.2742	0.2883071
variety:field	10	1.3574	0.13574	4.8330	0.0001022 ***
Residuals	45	1.2639	0.02809		

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

y <- alfalfa$yield
y... <- predict(lm(yield ~ 1,data = alfalfa))
yi.. <- predict(lm(yield ~ variety,data = alfalfa))
y.j. <- predict(lm(yield ~ date,data = alfalfa))
y..k <- predict(lm(yield ~ field,data = alfalfa))
yij. <- predict(lm(yield ~ variety + date + variety:date,data = alfalfa))
yi.k <- predict(lm(yield ~ variety + field + variety:field,data = alfalfa))

SST <- sum((y - y...)^2)
SSA <- sum((yi.. - y...)^2)
SSC <- sum((y..k - y...)^2)
SSAC <- sum((yi.k - (yi.. + y..k - y...))^2)
SSB <- sum((y.j. - y...)^2)
SSAB <- sum((yij. - (yi.. + y.j. - y...))^2)
SSE <- SST - (SSA + SSC + SSAC + SSB + SSAB)

```

```

a <- 3
b <- 4
c <- 6

MSA <- SSA / (a-1)
MSC <- SSC / (c-1)
MSAC <- SSAC / ((c-1)*(a-1))
MSB <- SSB / (b-1)
MSAB <- SSAB / ((a-1)*(b-1))
MSE <- SSE / (a*(b-1)*(c-1))

FA_incorrect <- MSA / MSE
FC_incorrect <- MSC / MSE
FA <- MSA / MSAC
FC <- MSC / MSAC
FAC <- MSAC / MSE
FB <- MSB / MSE
FAB <- MSAB / MSE

pA_incorrect <- 1 - pf(FA_incorrect, a-1, a*(b-1)*(c-1))
pC_incorrect <- 1 - pf(FC_incorrect, c-1, a*(b-1)*(c-1))
pA <- 1 - pf(FA, a-1, (c-1)*(a-1))
pC <- 1 - pf(FC, c-1, (c-1)*(a-1))
pAC <- 1 - pf(FAC, (c-1)*(a-1), a*(b-1)*(c-1))
pB <- 1 - pf(FB, b-1, a*(b-1)*(c-1))
pAB <- 1 - pf(FAB, (a-1)*(b-1), a*(b-1)*(c-1))

```

Correct ANOVA table:

Source	Df	SS	MS	F value	p value
A	2	0.175	0.088	0.646	0.5449
C	5	4.139	0.828	6.098	0.0076
AC	10	1.357	0.136	4.833	0.0001
B	3	1.973	0.658	23.412	0.0000
AB	6	0.215	0.036	1.274	0.2883
Error	45	1.264	0.028		
Total	71	9.123			

Correct p values for fixed effects with lmerTest package:

```
library(lmerTest) # first time run install.packages("lmerTest")
lmer_out <- lmer(yield ~ variety + date + variety:date + (1|field) + (1|field:variety),
                data = alfalfa)
anova(lmer_out)
```

Type III Analysis of Variance Table with Satterthwaite's method

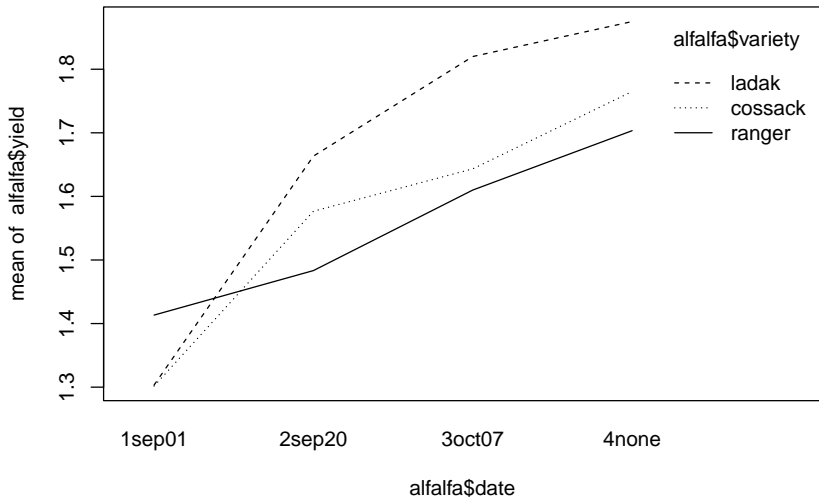
	Sum Sq	Mean Sq	NumDF	DenDF	F value	Pr(>F)
variety	0.03626	0.01813	2	10	0.6455	0.5449
date	1.97274	0.65758	3	45	23.4123	2.789e-09 ***
variety:date	0.21472	0.03579	6	45	1.2742	0.2883

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Rescales SS_A and MS_A by the factor MS_{Error} / MS_{AC} .

Note: If $\hat{\sigma}_C^2 = 0$, it will compute the whole-plot effect p value differently!

```
interaction.plot(alfalfa$date,alfalfa$variety,alfalfa$yield)
```



MoMs for variance components in RCB split-plot design

The method of moments estimators for σ_C^2 , σ_{AC}^2 , and σ_ε^2 are

$$\blacktriangleright \dot{\sigma}_\varepsilon^2 = \text{MS}_{\text{Error}}$$

$$\blacktriangleright \dot{\sigma}_{AC}^2 = \frac{\text{MS}_{AC} - \text{MS}_{\text{Error}}}{b}$$

$$\blacktriangleright \dot{\sigma}_C^2 = \frac{\text{MS}_C - \text{MS}_{AC}}{ab}$$

It is possible to obtain negative values for $\dot{\sigma}_{AC}^2$ and $\dot{\sigma}_C^2$. Use REML.

Alfalfa data (cont)

Obtain REML estimates of σ_C^2 , σ_{AC}^2 , and σ_ε^2 from `lmer()`.

```
summary(lmer_out)$varcor
```

Groups	Name	Std.Dev.
field:variety	(Intercept)	0.16406
field	(Intercept)	0.24014
Residual		0.16759

Variances of some means and difference in means

Contrast	Variance	MoM variance estimator
$\bar{Y}_{i..} - \bar{Y}_{i'..}$	$\frac{2}{bc}(b\sigma_{AC}^2 + \sigma_{\varepsilon}^2)$	$\frac{2}{bc} \text{MS}_{AC}$
$\bar{Y}_{.j} - \bar{Y}_{.j'}$	$\frac{2}{ac}\sigma_{\varepsilon}^2$	$\frac{2}{ac} \text{MS}_{\text{Error}}$
$\bar{Y}_{ij.} - \bar{Y}_{i'j.}$	$\frac{2}{c}(\sigma_{AC}^2 + \sigma_{\varepsilon}^2)$	$\frac{2}{c}[\text{MS}_{AC} + (b-1) \text{MS}_{\text{Error}}]$
$\bar{Y}_{ij.} - \bar{Y}_{ij'}$	$\frac{2}{c}\sigma_{\varepsilon}^2$	$\frac{2}{c} \text{MS}_{\text{Error}}$
$\bar{Y}_{ij.} - \bar{Y}_{i'j'}$	$\frac{2}{c}(\sigma_{AC}^2 + \sigma_{\varepsilon}^2)$	$\frac{2}{c}[\text{MS}_{AC} + (b-1) \text{MS}_{\text{Error}}]$
$\bar{Y}_{i..}$	$\frac{1}{bc}[b\sigma_C^2 + b\sigma_{AC}^2 + \sigma_{\varepsilon}^2]$	$\frac{1}{abc}[\text{MS}_C + (a-1) \text{MS}_{AC}]$
$\bar{Y}_{.j}$	$\frac{1}{bc}[a\sigma_C^2 + \sigma_{AC}^2 + \sigma_{\varepsilon}^2]$	$\frac{1}{abc}[\text{MS}_C + (b-1) \text{MS}_{AC}]$
$\bar{Y}_{ij.}$	$\frac{1}{c}[\sigma_C^2 + \sigma_{AC}^2 + \sigma_{\varepsilon}^2]$	$\frac{1}{abc}[\text{MS}_C + (a-1) \text{MS}_{AC} + a(b-1) \text{MS}_{\text{Error}}]$

Some (unadjusted) CIs in RCB split plot design

Target	$(1 - \alpha)100\%$ confidence interval
$\bar{\mu}_{i.} - \bar{\mu}_{i'.$	$\bar{Y}_{i..} - \bar{Y}_{i'..} \pm t_{(a-1)(c-1), \alpha/2} \sqrt{MS_{AC}} \sqrt{\frac{2}{bc}}$
$\bar{\mu}_{.j} - \bar{\mu}_{.j'}$	$\bar{Y}_{.j.} - \bar{Y}_{.j'.} \pm t_{a(b-1)(c-1), \alpha/2} \sqrt{MS_{Error}} \sqrt{\frac{2}{ac}}$
$\mu_{ij} - \mu_{i'j}$	$\bar{Y}_{ij.} - \bar{Y}_{i'j.} \pm t_{\nu^*, \alpha/2} \sqrt{MS_{AC} + (b-1)MS_{Error}} \sqrt{\frac{2}{c}}$
$\mu_{ij} - \mu_{ij'}$	$\bar{Y}_{ij.} - \bar{Y}_{ij'.} \pm t_{a(b-1)(c-1), \alpha/2} \sqrt{MS_{Error}} \sqrt{\frac{2}{c}}$
$\mu_{ij} - \mu_{i'j'}$	$\bar{Y}_{ij.} - \bar{Y}_{i'j'.} \pm t_{\nu^*, \alpha/2} \sqrt{MS_{AC} + (b-1)MS_{Error}} \sqrt{\frac{2}{c}}$

In the above $\nu^* = \frac{MS_{AC} + (b-1)MS_{Error}}{\frac{MS_{AC}^2}{(a-1)(c-1)} + \frac{(b-1)^2 MS_{Error}^2}{a(b-1)(c-1)}}$ à la Satterthwaite¹.

¹a degrees of freedom approximation when one has not exactly a t-distribution.

Alfa data (cont)

Dunnett's comparison of marginal date means with no cutting as baseline:

$$\bar{Y}_{.j.} - \bar{Y}_{.b.} \pm d_{b-1, a(b-1)(c-1), \alpha} \sqrt{\text{MS}_{\text{Error}}} \sqrt{\frac{2}{ac}}$$

Replace, in previous slide, $t_{a(b-1)(c-1), \alpha/2}$ with $d_{b-1, a(b-1)(c-1), \alpha}$.

In Dunnett's table we cannot find $d_{4,45,0.05}$, so take $d_{4,40,0.05} = 2.44$.

Table A.5 Critical Values for Dunnett's Two-Sided Test of Treatments versus Control.

Error df	Two-sided α	T = Number of Groups Counting Both Treatments and Control						
		2	3	4	5	6	7	8
5	0.05	2.57	3.03	3.29	3.48	3.62	3.73	3.82
5	0.01	4.03	4.63	4.97	5.22	5.41	5.56	5.68
6	0.05	2.45	2.86	3.10	3.26	3.39	3.49	3.57
6	0.01	3.71	4.21	4.51	4.71	4.87	5.00	5.10
7	0.05	2.36	2.75	2.97	3.12	3.24	3.33	3.41
7	0.01	3.50	3.95	4.21	4.39	4.53	4.64	4.74
8	0.05	2.31	2.67	2.88	3.02	3.13	3.22	3.29
8	0.01	3.36	3.77	4.00	4.17	4.29	4.40	4.48
9	0.05	2.26	2.61	2.81	2.95	3.05	3.14	3.20
9	0.01	3.25	3.63	3.85	4.01	4.12	4.22	4.30
10	0.05	2.23	2.57	2.76	2.89	2.99	3.07	3.14
10	0.01	3.17	3.53	3.74	3.88	3.99	4.08	4.16
11	0.05	2.20	2.53	2.72	2.84	2.94	3.02	3.08
11	0.01	3.11	3.45	3.65	3.79	3.89	3.98	4.05
12	0.05	2.18	2.50	2.68	2.81	2.90	2.98	3.04
12	0.01	3.05	3.39	3.58	3.71	3.81	3.89	3.96
13	0.05	2.16	2.48	2.65	2.78	2.87	2.94	3.00
13	0.01	3.01	3.33	3.52	3.65	3.74	3.82	3.89
14	0.05	2.14	2.46	2.63	2.75	2.84	2.91	2.97
14	0.01	2.98	3.29	3.47	3.59	3.69	3.76	3.83
15	0.05	2.13	2.44	2.61	2.73	2.82	2.89	2.95
15	0.01	2.95	3.25	3.43	3.55	3.64	3.71	3.78
16	0.05	2.12	2.42	2.59	2.71	2.80	2.87	2.92
16	0.01	2.92	3.22	3.39	3.51	3.60	3.67	3.73
17	0.05	2.11	2.41	2.58	2.69	2.78	2.85	2.90
17	0.01	2.90	3.19	3.36	3.47	3.56	3.63	3.69
18	0.05	2.10	2.40	2.56	2.68	2.76	2.83	2.89
18	0.01	2.88	3.17	3.33	3.44	3.53	3.60	3.66
19	0.05	2.09	2.39	2.55	2.66	2.75	2.81	2.87
19	0.01	2.86	3.15	3.31	3.42	3.50	3.57	3.63
20	0.05	2.09	2.38	2.54	2.65	2.73	2.80	2.86
20	0.01	2.85	3.13	3.29	3.40	3.48	3.55	3.60
25	0.05	2.06	2.34	2.50	2.61	2.69	2.75	2.81
25	0.01	2.79	3.06	3.21	3.31	3.39	3.45	3.51
30	0.05	2.04	2.32	2.47	2.58	2.66	2.72	2.77
30	0.01	2.75	3.01	3.15	3.25	3.33	3.39	3.44
40	0.05	2.02	2.29	2.44	2.54	2.62	2.68	2.73
40	0.01	2.70	2.95	3.09	3.19	3.26	3.32	3.37
60	0.05	2.00	2.27	2.41	2.51	2.58	2.64	2.69
60	0.01	2.66	2.90	3.03	3.12	3.19	3.25	3.29

This table produced from the SAS System using function PROBMCC(DUNNETT2,1 - α ,df,k), where $k = T - 1$.

Figure 1: Table A.5 from Mohr, Wilson, and Freund (2021)

```

y.1.bar <- mean(alfalfa$yield[alfalfa$date=="1sep01"])
y.2.bar <- mean(alfalfa$yield[alfalfa$date=="2sep20"])
y.3.bar <- mean(alfalfa$yield[alfalfa$date=="3oct07"])
y.4.bar <- mean(alfalfa$yield[alfalfa$date=="4none"])

se <- sqrt(MSE) * sqrt(2/(a*c))
me <- 2.44 * se

dtab <- rbind(c(y.1.bar - y.4.bar - me, y.1.bar - y.4.bar + me),
             c(y.2.bar - y.4.bar - me, y.2.bar - y.4.bar + me),
             c(y.3.bar - y.4.bar - me, y.3.bar - y.4.bar + me))

colnames(dtab) <- c("lower", "upper")
rownames(dtab) <- c("1sep01 - none",
                  "2sep20 - none",
                  "3oct07 - none")

round(dtab, 3)

```

```

          lower upper
1sep01 - none -0.578 -0.305
2sep20 - none -0.343 -0.070
3oct07 - none -0.226  0.046

```

Unadjusted CIs with `ls_means()` from R package `lmerTest`:

```
ls_means(lmer_out, which="date")
```

Least Squares Means table:

	Estimate	Std. Error	df	t value	lower	upper	Pr(> t)	
date1sep01	1.33944	0.11255	6.1	11.901	1.06474	1.61415	1.977e-05	***
date2sep20	1.57444	0.11255	6.1	13.989	1.29974	1.84915	7.646e-06	***
date3oct07	1.69111	0.11255	6.1	15.026	1.41641	1.96581	5.010e-06	***
date4none	1.78111	0.11255	6.1	15.825	1.50641	2.05581	3.684e-06	***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Confidence level: 95%

Degrees of freedom method: Satterthwaite

```
ls_means(lmer_out, pairwise = TRUE, which = "date")
```

Least Squares Means table:

	Estimate	Std. Error	df	t value	lower	upper
date1sep01 - date2sep20	-0.2350000	0.0558639	45	-4.2067	-0.3475156	-0.1224844
date1sep01 - date3oct07	-0.3516667	0.0558639	45	-6.2951	-0.4641823	-0.2391511
date1sep01 - date4none	-0.4416667	0.0558639	45	-7.9061	-0.5541823	-0.3291511
date2sep20 - date3oct07	-0.1166667	0.0558639	45	-2.0884	-0.2291823	-0.0041511
date2sep20 - date4none	-0.2066667	0.0558639	45	-3.6995	-0.3191823	-0.0941511
date3oct07 - date4none	-0.0900000	0.0558639	45	-1.6111	-0.2025156	0.0225156

Pr(>|t|)

date1sep01 - date2sep20	0.0001219	***
date1sep01 - date3oct07	1.138e-07	***
date1sep01 - date4none	4.723e-10	***
date2sep20 - date3oct07	0.0424494	*
date2sep20 - date4none	0.0005859	***
date3oct07 - date4none	0.1141598	

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Confidence level: 95%

Degrees of freedom method: Satterthwaite

References

Mohr, Donna L, William J Wilson, and Rudolf J Freund. 2021.
Statistical Methods. Academic Press.