STAT 516 Lec 10

Split-plot design

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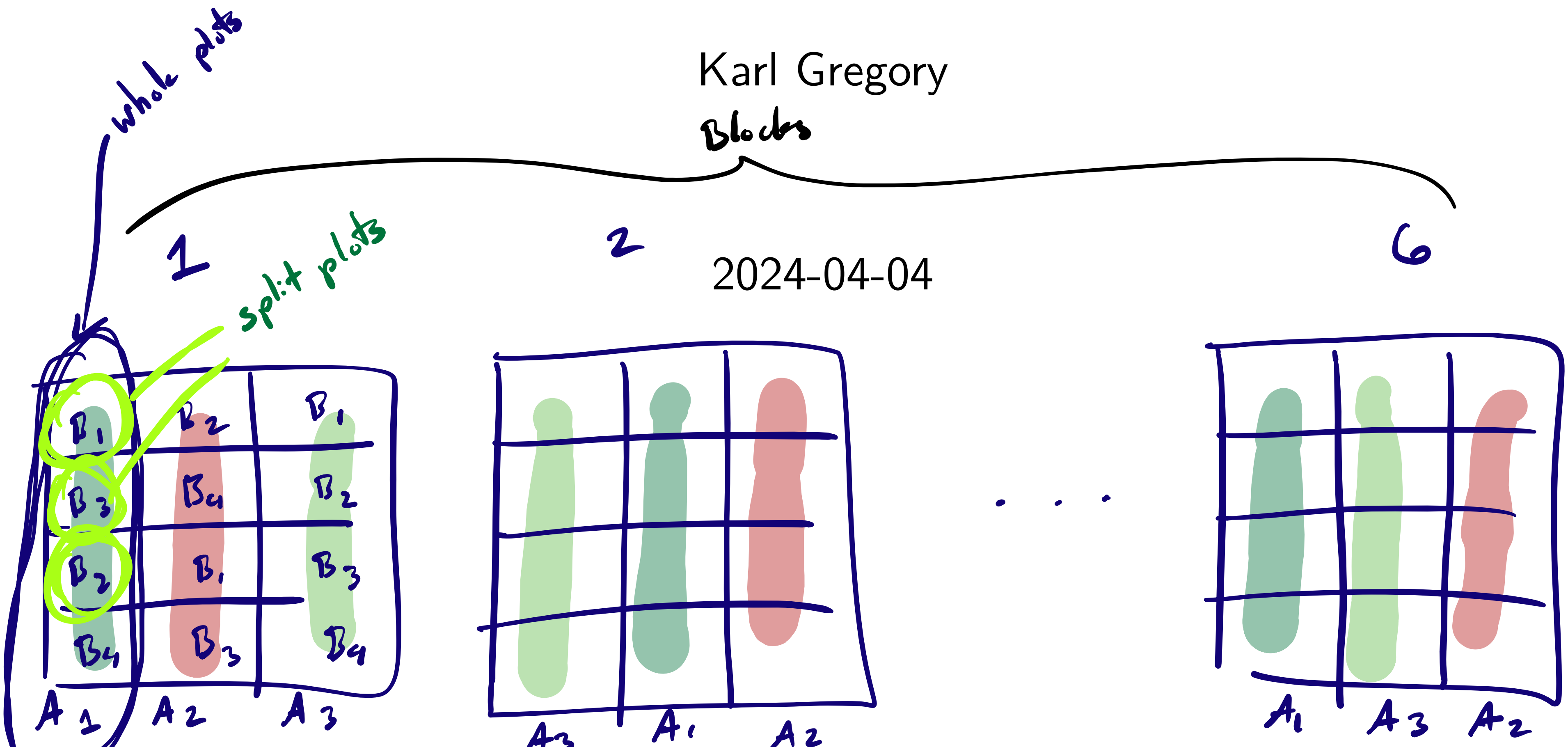


Alfalfa data from Dr. Longnecker's notes

- Six fields; three plots in each field; four sub-plots in each plot.
- Each plot randomly assigned a type of alfalfa.
- Each sub-plot randomly assigned a cutting date.
- Response for each subplot is yield in tons/acre in the following year.

Fields

Variety	Date	Blocks 1	2	3	4	5	6	TrT Mean $\bar{Y}_{ij.}$
Ladak	None	2.17	1.88	1.62	2.34	1.58	1.66	1.8750
	S1	1.58	1.26	1.22	1.59	1.25	0.94	1.3067
	S20	2.29	1.60	1.67	1.91	1.39	1.12	1.6633
	O7	2.23	2.01	1.82	2.10	1.66	1.10	1.8200
Cossack	None	2.33	2.01	1.70	1.78	1.42	1.35	1.7650
	S1	1.38	1.30	1.85	1.09	1.13	1.06	1.3017
	S20	1.86	1.70	1.81	1.54	1.67	0.88	1.5767
	O7	2.27	1.81	2.01	1.40	1.31	1.06	1.6433
Ranger	None	1.75	1.95	2.13	1.78	1.31	1.30	1.7033
	S1	1.52	1.47	1.80	1.37	1.01	1.31	1.4133
	S20	1.55	1.61	1.82	1.56	1.23	1.13	1.4833
	O7	1.56	1.72	1.99	1.55	1.51	1.33	1.6100

```r
alfalfa <- data.frame(yield = c(2.17,1.88,1.62,2.34,1.58,1.66,
                                1.56,1.26,1.22,1.59,1.25,0.94,
                                2.29,1.60,1.67,1.91,1.39,1.12,
                                2.23,2.01,1.82,2.10,1.66,1.10,
                                2.33,2.01,1.70,1.78,1.42,1.35,
                                1.38,1.30,1.85,1.09,1.13,1.06,
                                1.86,1.70,1.81,1.54,1.67,0.88,
                                2.27,1.81,2.01,1.40,1.31,1.06,
                                1.75,1.95,2.13,1.78,1.31,1.30,
                                1.52,1.47,1.80,1.37,1.01,1.31,
                                1.55,1.61,1.82,1.56,1.23,1.13,
                                1.56,1.72,1.99,1.55,1.51,1.33),
                      variety = as.factor(c(rep("ladak",24),
                                            rep("cossack",24),
                                            rep("ranger",24))),
                      date = as.factor(rep(c(rep("4none",6),
                                             rep("1sep01",6),
                                             rep("2sep20",6),
                                             rep("3oct07",6)),
                                           3)),
                      field = as.factor(rep(1:6,12)))
```

```r
head(alfalfa,n = 26)
```

```
   yield variety   date field
1   2.17   ladak  4none     1
2   1.88   ladak  4none     2
3   1.62   ladak  4none     3
4   2.34   ladak  4none     4
5   1.58   ladak  4none     5
6   1.66   ladak  4none     6
7   1.56   ladak 1sep01     1
8   1.26   ladak 1sep01     2
9   1.22   ladak 1sep01     3
10  1.59   ladak 1sep01     4
11  1.25   ladak 1sep01     5
12  0.94   ladak 1sep01     6
13  2.29   ladak 2sep20     1
14  1.60   ladak 2sep20     2
15  1.67   ladak 2sep20     3
16  1.91   ladak 2sep20     4
17  1.39   ladak 2sep20     5
18  1.12   ladak 2sep20     6
19  2.23   ladak 3oct07     1
20  2.01   ladak 3oct07     2
21  1.82   ladak 3oct07     3
22  2.10   ladak 3oct07     4
23  1.66   ladak 3oct07     5
24  1.10   ladak 3oct07     6
25  2.33 cossack  4none     1
26  2.01 cossack  4none     2
```

Randomized complete block split plot design

- Each EU randomly assigned to one level of whole-plot factor.
- Each EU receives all levels of split-plot factor in random order.
- Groups of EUs over which this is replicated are treated as blocks.

Treatment effects model for RCB split-plot design

Assume

$$Y_{ijk} = \boxed{\mu + \tau_i + \gamma_j + (\tau\gamma)_{ij}} + \boxed{C_k} + \boxed{(\tau C)_{ik}} + \varepsilon_{ijk},$$

two-way factorial; block effect $\sim N(0, \sigma_C^2)$; whole-plot effect $\sim N(0, \sigma_{AC}^2)$; $\sim N(0, \sigma_\varepsilon^2)$

for $i = 1, \ldots, a$, $j = 1, \ldots, b$ and $k = 1, \ldots, c$, where

- Y_{ijk} is the response of sub-plot j from whole plot i in block k.
- the τ_i are treatment effects for the whole-plot factor. (A)
- the γ_j are treatment effects for the split-plot factor. (B)
- the $(\tau\gamma)_{ij}$ are interaction effects between the factors.
- the C_k are independent Normal$(0, \sigma_C^2)$ block effects. (C)
- the $(\tau C)_{ik}$ are independent Normal$(0, \sigma_{AC}^2)$ whole-plot effects.
- the ε_{ijk} are independent Normal$(0, \sigma_\varepsilon^2)$ error terms.

Define the cell means as

$$\boxed{\mu_{ij} = \mu + \tau_i + \gamma_j + (\tau\gamma)_{ij},} \quad i = 1, \ldots, a, \quad j = 1, \ldots, b.$$

Sums of squares for the RCB split-plot design

SS	Symbol	Formula
Total	$\mathrm{SS}_{\mathrm{Tot}}$	$\sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}(Y_{ijk}-\bar{Y}_{...})^2$
A	SS_{A}	$bc\sum_{i=1}^{a}(\bar{Y}_{i..}-\bar{Y}_{...})^2$
C	SS_{C}	$ab\sum_{k=1}^{c}(\bar{Y}_{..k}-\bar{Y}_{...})^2$
AC	$\mathrm{SS}_{\mathrm{AC}}$	$b\sum_{i=1}^{a}\sum_{k=1}^{c}(\bar{Y}_{i.k}-(\bar{Y}_{i..}+\bar{Y}_{..k}-\bar{Y}_{...}))^2$
B	SS_{B}	$ac\sum_{j=1}^{b}(\bar{Y}_{.j.}-\bar{Y}_{...})^2$
AB	$\mathrm{SS}_{\mathrm{AB}}$	$c\sum_{i=1}^{a}\sum_{j=1}^{b}(Y_{ij.}-(\bar{Y}_{i..}+\bar{Y}_{.j.}-\bar{Y}_{...}))^2$
Error	$\mathrm{SS}_{\mathrm{Error}}$	$\mathrm{SS}_{\mathrm{Tot}}-(\mathrm{SS}_{\mathrm{A}}+\mathrm{SS}_{\mathrm{B}}+\mathrm{SS}_{\mathrm{AB}}+\mathrm{SS}_{\mathrm{C}}+\mathrm{SS}_{\mathrm{AC}})$

- We have $\mathrm{SS}_{\mathrm{Tot}}=\mathrm{SS}_{\mathrm{A}}+\mathrm{SS}_{\mathrm{B}}+\mathrm{SS}_{\mathrm{AB}}+\mathrm{SS}_{\mathrm{C}}+\mathrm{SS}_{\mathrm{AC}}$.

ANOVA table for RCB split-plot design

	Source	Df	SS	MS	F value
whole plot	A	$a-1$	SS_A	MS_A	$F_\text{A} = \text{MS}_\text{A} \,/\, \text{MS}_\text{AC}$
block	C	$c-1$	SS_C	MS_C	$F_\text{C} = \text{MS}_\text{C} \,/\, \text{MS}_\text{AC}$
whole plot effect	AC	$(a-1)(c-1)$	SS_AC	MS_AC	$F_\text{AC} = \text{MS}_\text{AC} \,/\, \text{MS}_\text{Error}$
split plot	B	$b-1$	SS_B	MS_B	$F_\text{B} = \text{MS}_\text{B} \,/\, \text{MS}_\text{Error}$
whole/split interaction	AB	$(a-1)(b-1)$	SS_AB	MS_AB	$F_\text{AB} = \text{MS}_\text{AB} \,/\, \text{MS}_\text{Error}$
	Error	$a(b-1)(c-1)$	SS_Error	MS_Error	
	Total	$abc-1$	SS_Tot		

1. Reject $H_0\colon \bar{\mu}_{1.} = \cdots = \bar{\mu}_{a.}$ if $F_\text{A} > F_{a-1,(a-1)(c-1),\alpha}$.
2. (block →) Reject $H_0\colon \sigma^2_C = 0$ if $F_\text{C} > F_{c-1,(a-1)(c-1),\alpha}$.
3. (whole plot →) Reject $H_0\colon \sigma^2_{AC} = 0$ if $F_\text{AC} > F_{(a-1)(c-1),a(b-1)(c-1),\alpha}$.
4. Reject $H_0\colon \bar{\mu}_{.1} = \cdots = \bar{\mu}_{.b}$ if $F_\text{B} > F_{b-1,a(b-1)(c-1),\alpha}$.
5. R. $H_0\colon \mu_{ij} = \bar{\mu}_{i.} + \bar{\mu}_{.j} - \bar{\mu}_{..}$ $\forall ij$ if $F_\text{AB} > F_{(a-1)(b-1),a(b-1)(c-1),\alpha}$.

Expected mean squares in RCB split plot design

$H_0: \bar{\mu}_{1.} = \cdots = \bar{\mu}_{a.} \Rightarrow \mathbb{E} MS_A = b\sigma^2_{AC} + \sigma^2_\varepsilon$ measures variability in $\bar{\mu}_{i.}$ values

Source	Df	Expected mean square
A	$a-1$	$bc\theta^2_A + b\sigma^2_{AC} + \sigma^2_\varepsilon$ $= \mathbb{E} MS_A$
C	$c-1$	$ab\sigma^2_C + b\sigma^2_{AC} + \sigma^2_\varepsilon$ $= \mathbb{E} MS_C$
AC	$(a-1)(c-1)$	$b\sigma^2_{AC} + \sigma^2_\varepsilon$
B	$b-1$	$ac\theta^2_B + \sigma^2_\varepsilon$
AB	$(a-1)(b-1)$	$c\theta^2_{AB} + \sigma^2_\varepsilon$
Error	$a(b-1)(c-1)$	σ^2_ε

$F_A = \frac{MS_A}{MS_{AC}}$

$F_B = \frac{MS_B}{MS_{Error}}$

In the above

- $\theta^2_A = (a-1)^{-1}\sum_{i=1}^a(\bar{\mu}_{i.} - \bar{\mu}_{..})^2$ $= 0$ if $\bar{\mu}_{1.} = \cdots = \bar{\mu}_{a.}$
- $\theta^2_B = (b-1)^{-1}\sum_{j=1}^b(\bar{\mu}_{.j} - \bar{\mu}_{..})^2$
- $\theta^2_{AB} = [(a-1)(b-1)]^{-1}\sum_{i=1}^a\sum_{j=1}^b(\mu_{ij} - (\bar{\mu}_{i.} + \bar{\mu}_{.j} - \bar{\mu}_{..}))^2$

Alfalfa data (cont)

The anova() function on the lm() output gives the wrong p-values.

```
lm_out <- lm(yield ~ variety + date + variety:date + field + field:variety,
             data = alfalfa)
anova(lm_out)
```

```
Analysis of Variance Table

Response: yield
              Df Sum Sq Mean Sq F value    Pr(>F)
variety        2 0.1753 0.08763  3.1198 0.0538447 .
date           3 1.9727 0.65758 23.4123 2.789e-09 ***
field          5 4.1388 0.82775 29.4710 3.798e-13 ***
variety:date   6 0.2147 0.03579  1.2742 0.2883071
variety:field 10 1.3574 0.13574  4.8330 0.0001022 ***
Residuals     45 1.2639 0.02809
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Compare to $F_{2,10,0.05} = 4.1$

p-value = 0.545

Fail to reject H_0: Variety has no effect.

$$F_{field} = \frac{.82775}{.13574} = 6.098.$$

Compare this to $F_{5,10,0.05}$

p-value = 0.0076

```r
y <- alfalfa$yield
y... <- predict(lm(yield ~ 1,data = alfalfa))
yi.. <- predict(lm(yield ~ variety,data = alfalfa))
y.j. <- predict(lm(yield ~ date,data = alfalfa))
y..k <- predict(lm(yield ~ field,data = alfalfa))
yij. <- predict(lm(yield ~ variety + date + variety:date,data = alfalfa))
yi.k <- predict(lm(yield ~ variety + field + variety:field,data = alfalfa))

SST <- sum((y - y...)^2)
SSA <- sum((yi.. - y...)^2)
SSC <- sum((y..k - y...)^2)
SSAC <- sum((yi.k - (yi.. + y..k - y...))^2)
SSB <- sum((y.j. - y...)^2)
SSAB <- sum((yij. - (yi.. + y.j. - y...))^2)
SSE <- SST - (SSA + SSC + SSAC + SSB + SSAB)
```

```r
a <- 3
b <- 4
c <- 6

MSA <- SSA / (a-1)
MSC <- SSC / (c-1)
MSAC <- SSAC / ((c-1)*(a-1))
MSB <- SSB / (b-1)
MSAB <- SSAB / ((a-1)*(b-1))
MSE <- SSE / (a*(b-1)*(c-1))

FA_incorrect <- MSA / MSE
FC_incorrect <- MSC / MSE
FA <- MSA / MSAC
FC <- MSC / MSAC
FAC <- MSAC / MSE
FB <- MSB / MSE
FAB <- MSAB / MSE

pA_incorrect <- 1 - pf(FA_incorrect,a-1,a*(b-1)*(c-1))
pC_incorrect <- 1 - pf(FC_incorrect,c-1,a*(b-1)*(c-1))
pA <- 1 - pf(FA,a-1,(c-1)*(a-1))
pC <- 1 - pf(FC,c-1,(c-1)*(a-1))
pAC  <- 1 - pf(FAC,(c-1)*(a-1),a*(b-1)*(c-1))
pB <- 1 - pf(FB,b-1,a*(b-1)*(c-1))
pAB <- 1 - pf(FAB,(a-1)*(b-1),a*(b-1)*(c-1))
```

Correct ANOVA table:

Source	Df	SS	MS	F value	p-value
A	2	0.175	0.088	0.646	0.5449
C	5	4.139	0.828	6.098	0.0076
AC	10	1.357	0.136	4.833	0.0001
B	3	1.973	0.658	23.412	0.0000
AB	6	0.215	0.036	1.274	0.2883
Error	45	1.264	0.028		
Total	71	9.123			

$Y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + C_k + (\tau C)_{ik} + \varepsilon_{ijk}$

Correct p values for fixed effects with `lmerTest` package:

```
library(lmerTest) # first time run install.packages("lmerTest")
lmer_out <- lmer(yield ~ variety + date + variety:date + (1|field) + (1|field:variety),
                 data = alfalfa)
anova(lmer_out)
```

Automatically Type III

```
Type III Analysis of Variance Table with Satterthwaite's method
              Sum Sq Mean Sq NumDF DenDF F value    Pr(>F)
variety      0.03626 0.01813     2    10  0.6455    0.5449
date         1.97274 0.65758     3    45 23.4123 2.789e-09 ***
variety:date 0.21472 0.03579     6    45  1.2742    0.2883
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

only fixed effects

correct p-value

same as anova(lm_out) p-values — already correct.

Rescales SS_A and MS_A by the factor $\text{MS}_\text{Error} / \text{MS}_\text{AC}$.

Obtain REML estimates of σ_C^2, σ_{AC}^2, and σ_ε^2 from `lmer()`.

```
library(lmerTest) # first time run install.packages("lmerTest")
lmer_out <- lmer(yield ~ variety + date + variety:date + (1|field) + (1|field:variety),
                 data = alfalfa)
summary(lmer_out)$varcor
```

```
 Groups        Name        Std.Dev.
 field:variety (Intercept) 0.16406
 field         (Intercept) 0.24014
 Residual                  0.16759
```

block

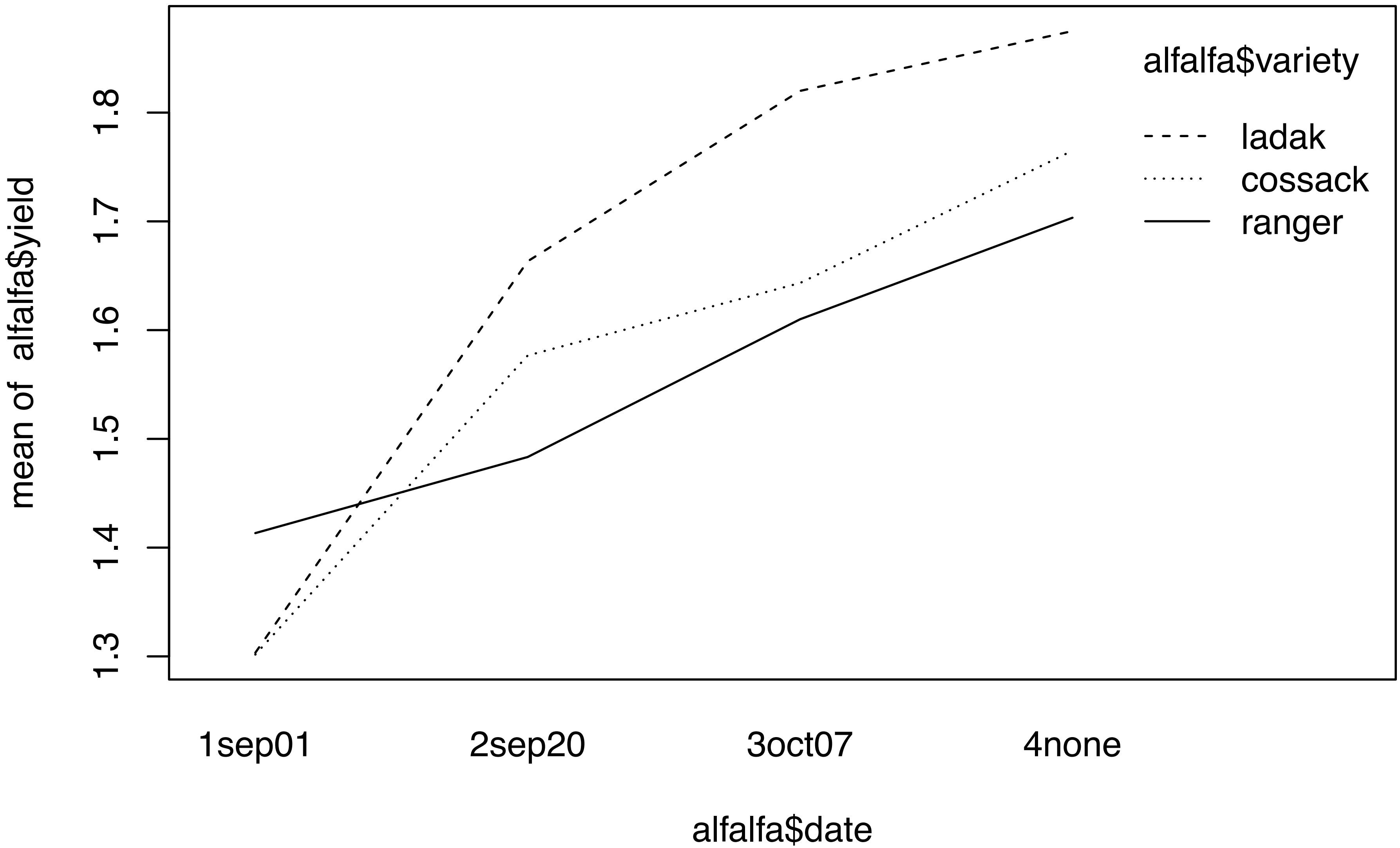
whole plot

$\hat{\sigma}_{AC}$

$\hat{\sigma}_C$

$\hat{\sigma}_\varepsilon$

```
interaction.plot(alfalfa$date,alfalfa$variety,alfalfa$yield)
```



MoMs estimators of variance components in the RCB split-plot design

The method of moments estimators for σ^2_C, σ^2_{AC}, and σ^2_ε are

- $\dot{\sigma}^2_\varepsilon = \mathrm{MS}_{\mathrm{Error}}$
- $\dot{\sigma}^2_{AC} = \dfrac{\mathrm{MS}_{\mathrm{AC}} - \mathrm{MS}_{\mathrm{Error}}}{b}$
- $\dot{\sigma}^2_C = \dfrac{\mathrm{MS}_{\mathrm{C}} - \mathrm{MS}_{\mathrm{AC}}}{ab}$

It is possible to obtain negative values for $\dot{\sigma}^2_{AC}$ and $\dot{\sigma}^2_C$. Use REML.

Variances of some means and difference in means

Contrast	Variance	MoM variance estimator
$\bar{Y}_{i..} - \bar{Y}_{i'..}$	$\frac{2}{bc}(b\sigma^2_{AC} + \sigma^2_{\varepsilon})$	$\frac{2}{bc}\,\mathrm{MS_{AC}}$
$\bar{Y}_{.j.} - \bar{Y}_{.j'.}$	$\frac{2}{ac}\sigma^2_{\varepsilon}$	$\frac{2}{ac}\,\mathrm{MS_{Error}}$
$\bar{Y}_{ij.} - \bar{Y}_{i'j.}$	$\frac{2}{c}(\sigma^2_{AC} + \sigma^2_{\varepsilon})$	$\frac{2}{c}[\mathrm{MS_{AC}} + (b-1)\,\mathrm{MS_{Error}}]$
$\bar{Y}_{ij.} - \bar{Y}_{ij'.}$	$\frac{2}{c}\sigma^2_{\varepsilon}$	$\frac{2}{c}\,\mathrm{MS_{Error}}$
$\bar{Y}_{ij.} - \bar{Y}_{i'j'.}$	$\frac{2}{c}(\sigma^2_{AC} + \sigma^2_{\varepsilon})$	$\frac{2}{c}[\mathrm{MS_{AC}} + (b-1)\,\mathrm{MS_{Error}}]$
$\bar{Y}_{i..}$	$\frac{1}{bc}[b\sigma^2_{C} + b\sigma^2_{AC} + \sigma^2_{\varepsilon}]$	$\frac{1}{abc}[\mathrm{MS_{C}} + (a-1)\,\mathrm{MS_{AC}}]$
$\bar{Y}_{.j.}$	$\frac{1}{bc}[a\sigma^2_{C} + \sigma^2_{AC} + \sigma^2_{\varepsilon}]$	$\frac{1}{abc}[\mathrm{MS_{C}} + (b-1)\,\mathrm{MS_{AC}}]$
$\bar{Y}_{ij.}$	$\frac{1}{c}[\sigma^2_{C} + \sigma^2_{AC} + \sigma^2_{\varepsilon}]$	$\frac{1}{abc}[\mathrm{MS_{C}} + (a-1)\,\mathrm{MS_{AC}} + a(b-1)\,\mathrm{MS_{Error}}]$

Some (unadjusted) CIs in RCB split plot design

Target	$(1-\alpha)100\%$ confidence interval
$\bar{\mu}_{i.} - \bar{\mu}_{i'.}$	$\bar{Y}_{i..} - \bar{Y}_{i'..} \pm t_{(a-1)(c-1),\alpha/2}\sqrt{\mathrm{MS}_{\mathrm{AC}}}\sqrt{\frac{2}{bc}}$
$\bar{\mu}_{.j} - \bar{\mu}_{.j'}$	$\bar{Y}_{.j.} - \bar{Y}_{.j'.} \pm t_{a(b-1)(c-1),\alpha/2}\sqrt{\mathrm{MS}_{\mathrm{Error}}}\sqrt{\frac{2}{ac}}$
$\mu_{ij} - \mu_{i'j}$	$\bar{Y}_{ij.} - \bar{Y}_{i'j.} \pm t_{\nu^*,\alpha/2}\sqrt{\mathrm{MS}_{\mathrm{AC}} + (b-1)\,\mathrm{MS}_{\mathrm{Error}}}\sqrt{\frac{2}{c}}$
$\mu_{ij} - \mu_{ij'}$	$\bar{Y}_{ij.} - \bar{Y}_{ij'.} \pm t_{a(b-1)(c-1),\alpha/2}\sqrt{\mathrm{MS}_{\mathrm{Error}}}\sqrt{\frac{2}{c}}$
$\mu_{ij} - \mu_{i'j'}$	$\bar{Y}_{ij.} - \bar{Y}_{i'j'.} \pm t_{\nu^*,\alpha/2}\sqrt{\mathrm{MS}_{\mathrm{AC}} + (b-1)\,\mathrm{MS}_{\mathrm{Error}}}\sqrt{\frac{2}{c}}$

In the above $\nu^* = \dfrac{\mathrm{MS}_{\mathrm{AC}} + (b-1)\,\mathrm{MS}_{\mathrm{Error}}}{\frac{\mathrm{MS}^2_{\mathrm{AC}}}{(a-1)(c-1)} + \frac{(b-1)^2\,\mathrm{MS}^2_{\mathrm{Error}}}{a(b-1)(c-1)}}$ à la Satterthwaite[1].

[1] a degrees of freedom approximation when one has not exactly a t-distribution.

Alfalfa data (cont)

Dunnett's comparison of marginal date means with no cutting as baseline:

$$\bar{Y}_{.j.} - \bar{Y}_{.b.} \pm d_{b-1,a(b-1)(c-1),\alpha} \sqrt{\mathrm{MS}_{\mathrm{Error}}} \sqrt{\frac{2}{ac}}$$

Replace, in previous slide, $t_{a(b-1)(c-1),\alpha/2}$ with $d_{b-1,a(b-1)(c-1),\alpha}$.

In Dunnett's table we cannot find $d_{4,45,0.05}$, so take $d_{4,40,0.05} = 2.44$.

Table A.5 Critical Values for Dunnett's Two-Sided Test of Treatments versus Control.

Error df	Two-sided α	T = Number of Groups Counting Both Treatments and Control						
		2	3	4	5	6	7	8
5	0.05	2.57	3.03	3.29	3.48	3.62	3.73	3.82
5	0.01	4.03	4.63	4.97	5.22	5.41	5.56	5.68
6	0.05	2.45	2.86	3.10	3.26	3.39	3.49	3.57
6	0.01	3.71	4.21	4.51	4.71	4.87	5.00	5.10
7	0.05	2.36	2.75	2.97	3.12	3.24	3.33	3.41
7	0.01	3.50	3.95	4.21	4.39	4.53	4.64	4.74
8	0.05	2.31	2.67	2.88	3.02	3.13	3.22	3.29
8	0.01	3.36	3.77	4.00	4.17	4.29	4.40	4.48
9	0.05	2.26	2.61	2.81	2.95	3.05	3.14	3.20
9	0.01	3.25	3.63	3.85	4.01	4.12	4.22	4.30
10	0.05	2.23	2.57	2.76	2.89	2.99	3.07	3.14
10	0.01	3.17	3.53	3.74	3.88	3.99	4.08	4.16
11	0.05	2.20	2.53	2.72	2.84	2.94	3.02	3.08
11	0.01	3.11	3.45	3.65	3.79	3.89	3.98	4.05
12	0.05	2.18	2.50	2.68	2.81	2.90	2.98	3.04
12	0.01	3.05	3.39	3.58	3.71	3.81	3.89	3.96
13	0.05	2.16	2.48	2.65	2.78	2.87	2.94	3.00
13	0.01	3.01	3.33	3.52	3.65	3.74	3.82	3.89
14	0.05	2.14	2.46	2.63	2.75	2.84	2.91	2.97
14	0.01	2.98	3.29	3.47	3.59	3.69	3.76	3.83
15	0.05	2.13	2.44	2.61	2.73	2.82	2.89	2.95
15	0.01	2.95	3.25	3.43	3.55	3.64	3.71	3.78
16	0.05	2.12	2.42	2.59	2.71	2.80	2.87	2.92
16	0.01	2.92	3.22	3.39	3.51	3.60	3.67	3.73
17	0.05	2.11	2.41	2.58	2.69	2.78	2.85	2.90
17	0.01	2.90	3.19	3.36	3.47	3.56	3.63	3.69
18	0.05	2.10	2.40	2.56	2.68	2.76	2.83	2.89
18	0.01	2.88	3.17	3.33	3.44	3.53	3.60	3.66
19	0.05	2.09	2.39	2.55	2.66	2.75	2.81	2.87
19	0.01	2.86	3.15	3.31	3.42	3.50	3.57	3.63
20	0.05	2.09	2.38	2.54	2.65	2.73	2.80	2.86
20	0.01	2.85	3.13	3.29	3.40	3.48	3.55	3.60
25	0.05	2.06	2.34	2.50	2.61	2.69	2.75	2.81
25	0.01	2.79	3.06	3.21	3.31	3.39	3.45	3.51
30	0.05	2.04	2.32	2.47	2.58	2.66	2.72	2.77
30	0.01	2.75	3.01	3.15	3.25	3.33	3.39	3.44
40	0.05	2.02	2.29	2.44	2.54	2.62	2.68	2.73
40	0.01	2.70	2.95	3.09	3.19	3.26	3.32	3.37
60	0.05	2.00	2.27	2.41	2.51	2.58	2.64	2.69
60	0.01	2.66	2.90	3.03	3.12	3.19	3.25	3.29

This table produced from the SAS System using function PROBMC('DUNNETT2',,$1-\alpha$,df,k), where $k = T - 1$.

Figure 1: Table A.5 from Mohr, Wilson, and Freund (2021)

```r
y.1.bar <- mean(alfalfa$yield[alfalfa$date=="1sep01"])
y.2.bar <- mean(alfalfa$yield[alfalfa$date=="2sep20"])
y.3.bar <- mean(alfalfa$yield[alfalfa$date=="3oct07"])
y.4.bar <- mean(alfalfa$yield[alfalfa$date=="4none"])

se <- sqrt(MSE) * sqrt(2/(a*c))
me <- 2.44 * se

dtab <- rbind(c(y.1.bar - y.4.bar - me,y.1.bar - y.4.bar + me),
              c(y.2.bar - y.4.bar - me,y.2.bar - y.4.bar + me),
              c(y.3.bar - y.4.bar - me,y.3.bar - y.4.bar + me))

colnames(dtab) <- c("lower","upper")
rownames(dtab) <- c("1sep01 - none",
                    "2sep20 - none",
                    "3oct07 - none")
round(dtab,3)
```

```
               lower  upper
1sep01 - none -0.578 -0.305
2sep20 - none -0.343 -0.070
3oct07 - none -0.226  0.046
```

Unadjusted CIs with `ls_means()` from R package `lmerTest`:

```r
ls_means(lmer_out,which="date")
```

← asks for marginal means of "date" factor

```
Least Squares Means table:

             Estimate Std. Error  df t value   lower   upper  Pr(>|t|)
date1sep01   1.33944    0.11255 6.1  11.901 1.06474 1.61415 1.977e-05 ***
date2sep20   1.57444    0.11255 6.1  13.989 1.29974 1.84915 7.646e-06 ***
date3oct07   1.69111    0.11255 6.1  15.026 1.41641 1.96581 5.010e-06 ***
date4none    1.78111    0.11255 6.1  15.825 1.50641 2.05581 3.684e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

  Confidence level: 95%
  Degrees of freedom method: Satterthwaite
```

add to get all pairwise differences.

```r
ls_means(lmer_out, pairwise = TRUE, which = "date")
```

```
Least Squares Means table:

                          Estimate Std. Error df t value      lower      upper
date1sep01 - date2sep20 -0.2350000  0.0558639 45 -4.2067 -0.3475156 -0.1224844
date1sep01 - date3oct07 -0.3516667  0.0558639 45 -6.2951 -0.4641823 -0.2391511
date1sep01 - date4none  -0.4416667  0.0558639 45 -7.9061 -0.5541823 -0.3291511
date2sep20 - date3oct07 -0.1166667  0.0558639 45 -2.0884 -0.2291823 -0.0041511
date2sep20 - date4none  -0.2066667  0.0558639 45 -3.6995 -0.3191823 -0.0941511
date3oct07 - date4none  -0.0900000  0.0558639 45 -1.6111 -0.2025156  0.0225156
                         Pr(>|t|)
date1sep01 - date2sep20 0.0001219 ***
date1sep01 - date3oct07 1.138e-07 ***
date1sep01 - date4none  4.723e-10 ***
date2sep20 - date3oct07 0.0424494 *
date2sep20 - date4none  0.0005859 ***
date3oct07 - date4none  0.1141598
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

  Confidence level: 95%
  Degrees of freedom method: Satterthwaite
```

References

Mohr, Donna L, William J Wilson, and Rudolf J Freund. 2021. *Statistical Methods*. Academic Press.