

# STAT 516 Lec 11

Analysis of covariance (ANCOVA)

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# VO<sub>2</sub> max data from Kuehl (2000)

Change in VO<sub>2</sub> max of <sup>EVs</sup> 12 males (after-minus-before) randomly assigned to two exercise programs (running, step aerobics). Ages recorded.

<i>Group</i>	<i>Age</i>	<i>Change</i>	<i>Group</i>	<i>Age</i>	<i>Change</i>
Aerobic	31	17.05	Running	23	-0.87
	23	4.96		22	-10.74
	27	10.40		22	-3.27
	28	11.05		25	-1.97
	22	0.26		27	7.50
	24	2.51		20	-7.25
Mean	25.83	7.71		23.17	-2.77
Std. Err.	1.40	2.55		1.01	2.54

Source: D. Allen, Exercise Physiology, University of Arizona.

Which exercise program lead to a greater average change in VO<sub>2</sub> max?

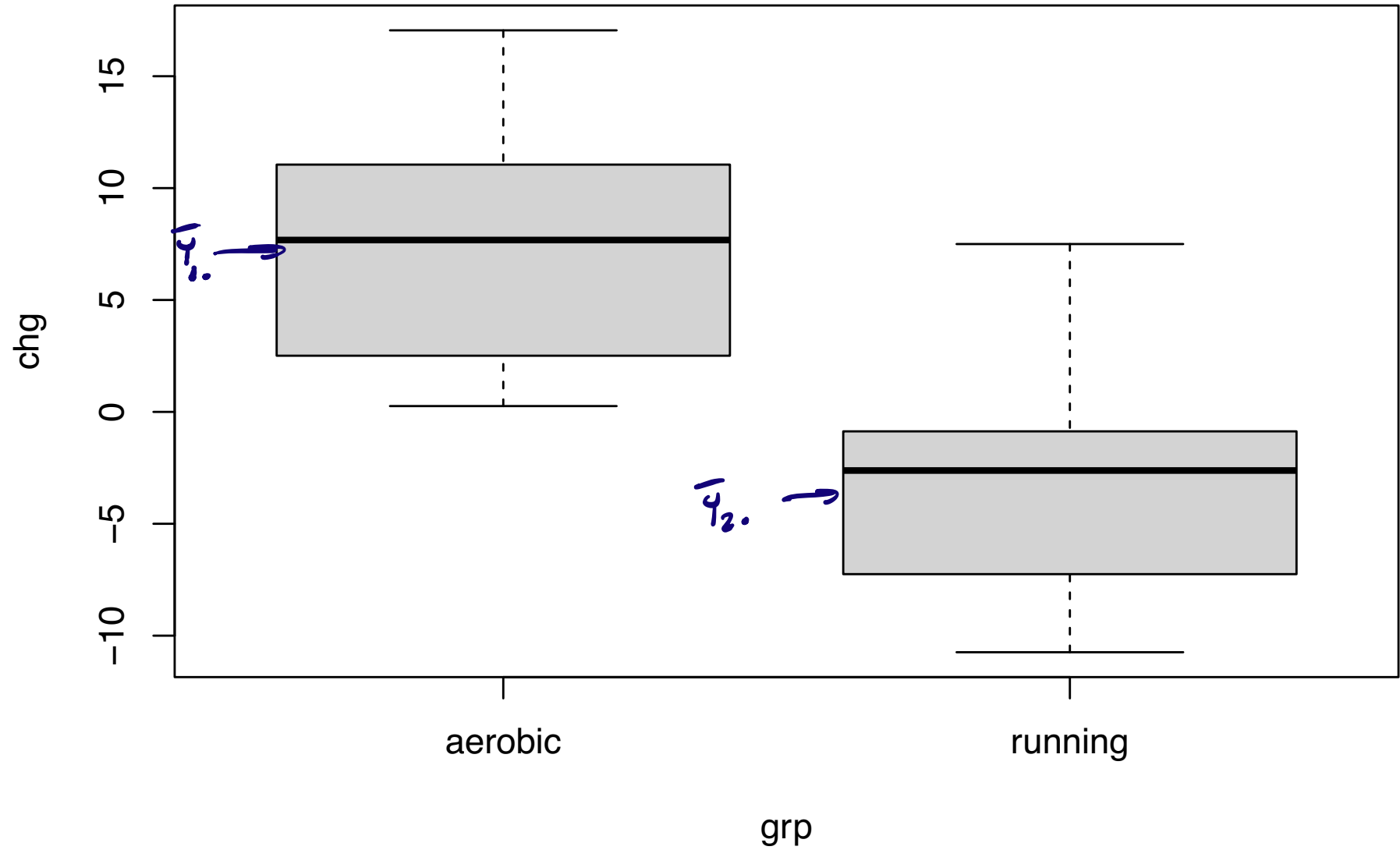
What role does age play?

```
vo2max <- data.frame(chg = c(17.05,4.96,10.40,11.05,0.26,2.51,  
                           -0.87,-10.74,-3.27,-1.97,7.50,-7.25),  
                    grp = as.factor(c(rep("aerobic",6),rep("running",6))),  
                    age = c(31,23,27,28,22,24,23,22,22,25,27,20))
```

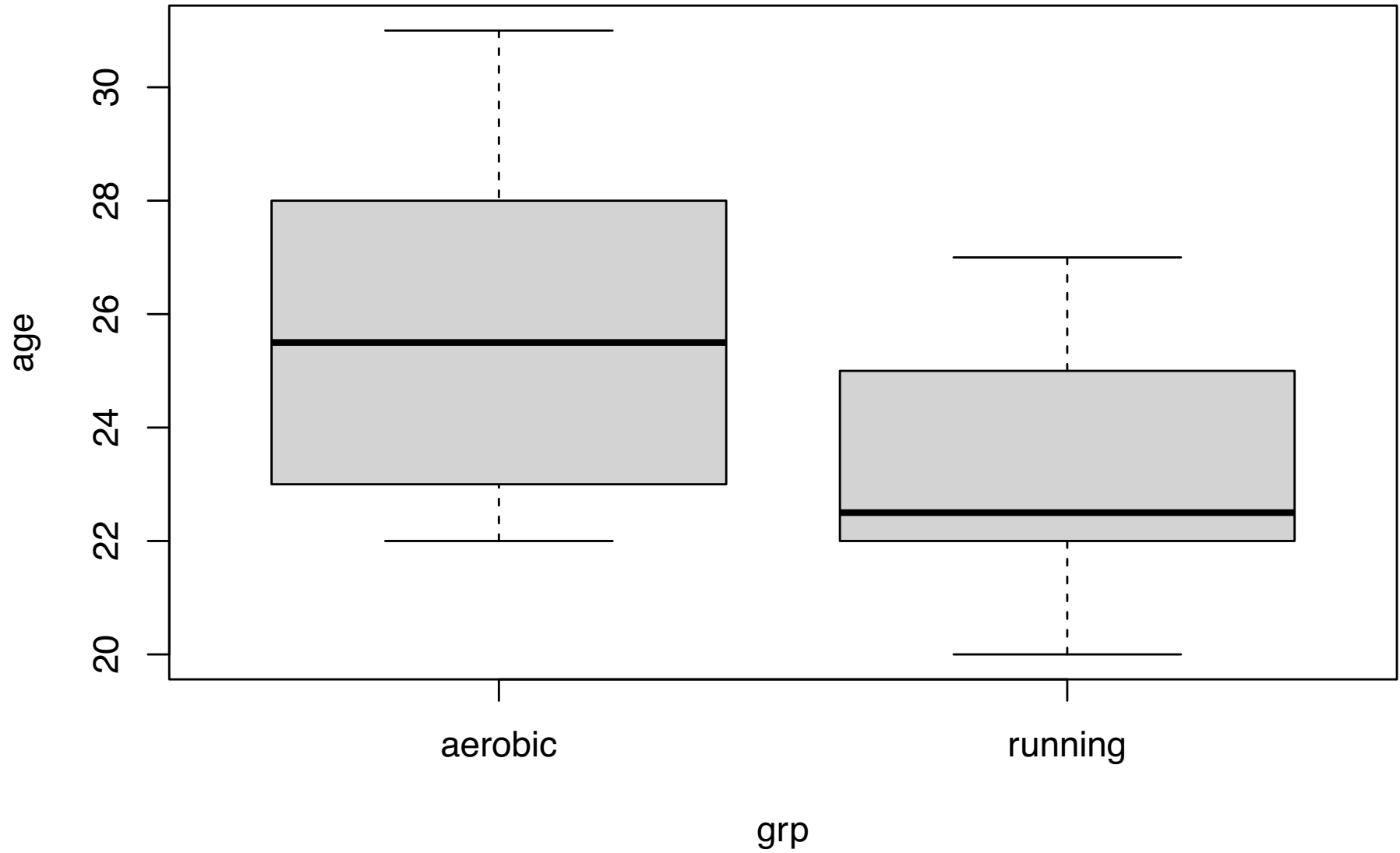
vo2max

	chg	grp	age
1	17.05	aerobic	31
2	4.96	aerobic	23
3	10.40	aerobic	27
4	11.05	aerobic	28
5	0.26	aerobic	22
6	2.51	aerobic	24
7	-0.87	running	23
8	-10.74	running	22
9	-3.27	running	22
10	-1.97	running	25
11	7.50	running	27
12	-7.25	running	20

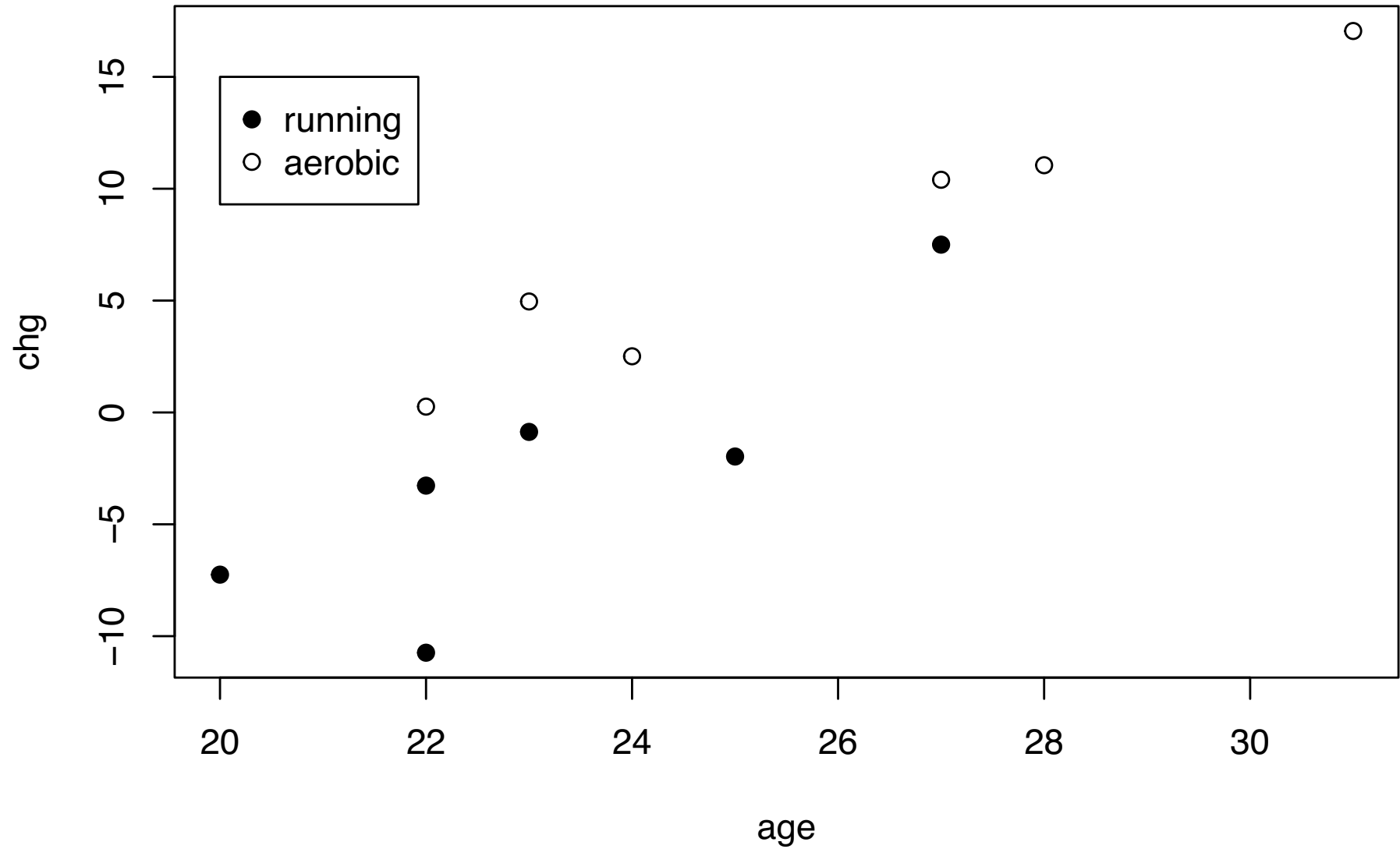
```
boxplot(chg ~ grp, data = vo2max)
```



```
boxplot(age ~ grp, data = vo2max)
```



```
plot(chg ~ age, pch = ifelse(grp == "running",19,1), data = vo2max)
legend(x = 20,y = 15,legend = c("running","aerobic"), pch = c(19,1))
```



# Analysis of covariance

- ▶ Useful when EUs are not homogeneous.
- ▶ Measurement capturing EU inhomogeneity is called a covariate.
- ▶ Can isolate treatment effects even when EUs differ across treatment groups.

age  
↓

covariate.

# Single-slope analysis of covariance (ANCOVA) model

Example:  $i=1,2$  ( $a=2$ )  
 $j=1,\dots,6$  ( $n_1=n_2=6$ )

Assume

$$Y_{ij} = \mu + \tau_i + \beta x_{ij} + \varepsilon_{ij}$$

Diagram annotations:  
- A box around  $\mu + \tau_i$  is labeled  $\mu_i$ .  
- An arrow points from  $\tau_i$  to the text "treatment effect".  
- An arrow points from  $\beta x_{ij}$  to the text "covariate effect".

for  $i = 1, \dots, a$ ,  $j = 1, \dots, n_i$ , where

- ▶  $Y_{ij}$  is the response of EU  $j$  in treatment group  $i$ .
- ▶  $\mu$  is a baseline or overall mean.
- ▶ the  $\tau_i$  are treatment effects.
- ▶ the  $x_{ij}$  are covariate values measured on the EUs.
- ▶  $\beta$  is a slope coefficient expressing the effect of the covariate.
- ▶ the  $\varepsilon_{ij}$  are independent  $\text{Normal}(0, \sigma_\varepsilon^2)$  error terms.
- ▶  $n_1 + \dots + n_a = N$ . Unbalancedness not an issue.

Set  $\mu_i = \mu + \tau_i$  for  $i = 1, \dots, a$ .



# Goals in analysis of covariance

- ✓ 1. Estimate the parameters  $\mu, \tau_1, \dots, \tau_a,$  and  $\beta$ .
- ✓ 2. Visualize the data.
- ✓ 3. Fit full and reduced models and collect error sums of squares.
- ✓ 4. Test whether the covariate has any effect.
- ✓ 5. Test for a treatment effect.
- ✓ 6. Adjust treatment group means for the inhomogeneity of the EUs.
- ✓ 7. Compare the adjusted group means.

fitted values:

$$\hat{Y}_{ij} = \hat{\mu} + \hat{\tau}_i + \hat{\beta} x_{ij}$$

# Parameter constraints

$$Y_{ij} = \underbrace{\mu + \tau_i}_{a} + \underbrace{\beta x_{ij}}_1 + \epsilon_{ij}$$

- ▶ There are  $a$  treatment groups and one covariate slope.
- ▶ The model has  $a + 2$  parameters, which is one too-many.
- ▶ R will set  $\tau_1 = 0$  so that it can estimate all the parameters.

# VO<sub>2</sub> max data (cont)

```
lm_out <- lm(chg ~ grp + age, data = vo2max)
summary(lm_out)
```

running vs aerobic  
↓  
"factor"  
"numeric"

Call:

```
lm(formula = chg ~ grp + age, data = vo2max)
```

Residuals:

Min	1Q	Median	3Q	Max
-5.7731	-0.9902	0.1395	1.8254	3.0374

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	-41.0139	7.7144	-5.317	0.000483 ***
grprunning	-5.4426	1.7965	-3.030	0.014255 *
age	1.8859	0.2953	6.386	0.000127 ***

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.797 on 9 degrees of freedom  
Multiple R-squared: 0.902, Adjusted R-squared: 0.8802  
F-statistic: 41.42 on 2 and 9 DF, p-value: 2.887e-05

$\hat{\tau}_{\text{running}}$   
( $\hat{\tau}_{\text{aerobic}} = 0$ )

$\hat{\beta}$

```
parms <- coef(lm_out)
parms
```

```
(Intercept)  grprunning  age
-41.013882   -5.442621    1.885892
```

$\hat{\beta}$

$\hat{\beta}_{\text{running}}$

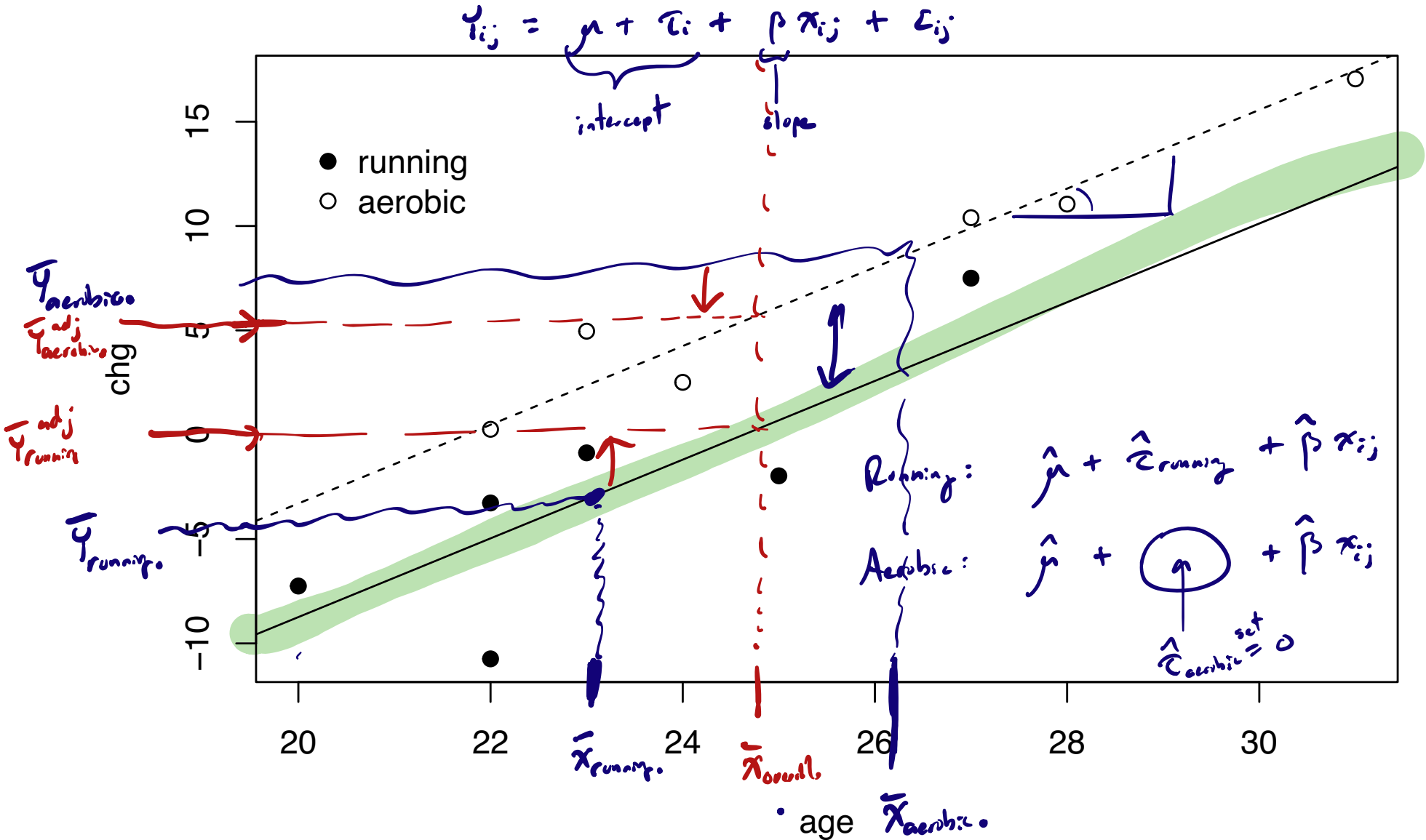
$\hat{\beta}$

$\text{abline}(\text{intercept}, \text{slope})$

```

plot(chg ~ age, pch = ifelse(grp == "running",19,1), data = vo2max)
abline(parms[1] + parms[2] * parms[3])
abline(parms[1],parms[3],lty = 2)
legend(x = 20,y = 15,legend = c("running","aerobic"), pch = c(19,1), bty = "n")

```



# Full and reduced models in ANCOVA

Want to test

1.  $H_0: \beta = 0$

vs

$H_1: \beta \neq 0$

2.  $H_0: \mu_1 = \dots = \mu_a$

vs

$H_1: \text{Not all}$

treatment means equal.

We construct test statistics by fitting full and reduced models:

- ▶ Full model:  $Y_{ij} = \mu + \tau_i + \beta x_{ij} + \varepsilon_{ij}$
- ▶ Reduced model (no covariate effect):  $Y_{ij} = \mu + \tau_i + \varepsilon_{ij}$
- ▶ Reduced model (no treatment effect):  $Y_{ij} = \mu + \beta x_{ij} + \varepsilon_{ij}$

↑  
Same intercept  
for all groups

# Full- and reduced-model error sums of squares

SS	Formula
$SS_{\text{Error}}(\text{Full})$	$\sum_{i=1}^a \sum_{j=1}^{n_{ij}} (Y_{ij} - (\hat{\mu} + \hat{\tau}_i + \hat{\beta}x_{ij}))^2$
$SS_{\text{Error}}(\text{No covariate})$	$\sum_{i=1}^a \sum_{j=1}^{n_{ij}} (Y_{ij} - (\hat{\mu} + \hat{\tau}_i))^2$
$SS_{\text{Error}}(\text{No treatment})$	$\sum_{i=1}^a \sum_{j=1}^{n_{ij}} (Y_{ij} - (\hat{\mu} + \hat{\beta}x_{ij}))^2$

Now define these difference in error sums of squares:

$$\begin{aligned} \blacktriangleright \boxed{SS_{\text{Cov}}} &= SS_{\text{Error}}(\text{No covariate}) - SS_{\text{Error}}(\text{Full}) \\ \blacktriangleright \boxed{SS_{\text{Trt}}} &= SS_{\text{Error}}(\text{No treatment}) - SS_{\text{Error}}(\text{Full}) \end{aligned}$$

$\swarrow$  Type III sums of squares

# ANCOVA table

$\beta$

Source	Df	SS	MS	F value
Covariate	1	$SS_{\text{Cov}}$	$MS_{\text{Cov}}$	$F_{\text{Cov}} = MS_{\text{Cov}} / MS_{\text{Error}}$
Treatment	$a - 1$	$SS_{\text{Trt}}$	$MS_{\text{Trt}}$	$F_{\text{Trt}} = MS_{\text{Trt}} / MS_{\text{Error}}$
Error	$N - a - 1$	$SS_{\text{Error}}$	$MS_{\text{Error}}$	

1. Reject  $H_0: \beta = 0$  at  $\alpha$  if  $F_{\text{Cov}} > F_{1, N-a-1, \alpha}$ .

2. Reject  $H_0: \mu_1 = \dots = \mu_a$  at  $\alpha$  if  $F_{\text{Trt}} > F_{a-1, N-a-1, \alpha}$ .



# VO<sub>2</sub> max data (cont)

Using `anova()` on the `lm()` output gives the wrong SS (gives sequential).

Use `Anova()` from R package `car` on the `lm()` output.

```
library(car) # first time run install.packages("car")
# Use type = "II" or type = "III"
Anova(lm_out, type = "III")
```

Anova Table (Type III tests)

Response: chg

	Sum Sq	Df	F value	Pr(>F)
(Intercept)	221.06	1	28.2653	0.0004832 ***
grp	71.79	1	9.1788	0.0142548 *
age	318.91	1	40.7759	0.0001274 ***
Residuals	70.39	9		

← ignore p-value for  $H_0: \mu_1 = \dots = \mu_a$

↑ p-value for  $H_0: \beta = 0$ .

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

```

lm_nocov <- lm(chg ~ grp, data = vo2max) # reduced model with no covariate
lm_notrt <- lm(chg ~ age, data = vo2max) # reduced model with no treatment
lm_full <- lm(chg ~ age + grp, data = vo2max) # full model

SSE_nocov <- sum(lm_nocov$resid^2)
SSE_notrt <- sum(lm_notrt$resid^2)
SSE_full <- sum(lm_full$resid^2)

SSCov <- SSE_nocov - SSE_full
SSTrt <- SSE_notrt - SSE_full
SSE <- SSE_full

a <- 2
N <- nrow(vo2max)

MSCov <- SSCov / 1
MSTrt <- SSTrt / (a-1)
MSE <- SSE / (N - a - 1)

FCov <- MSCov / MSE
FTrt <- MSTrt / MSE

pCov <- 1 - pf(FCov, 1, N - a - 1)
pTrt <- 1 - pf(FTrt, a-1, N - a - 1)

```

# Treatment means in terms of parameter estimates

$$\bar{x}_{i.} = \frac{1}{n_i} \sum_{j=1}^{n_i} x_{ij} \quad \left( \begin{array}{l} \text{mean of covariate values} \\ \text{for group } i \end{array} \right)$$

- ▶ We can write the treatment group means as

$$\bar{Y}_{i.} = \hat{\mu} + \hat{\tau}_i + \hat{\beta} \bar{x}_{i.} \quad \text{for } i = 1, \dots, a,$$

*average age for group i*

- ▶ We also define the covariate-adjusted means

$$\bar{Y}_{i.}^{\text{adj}} = \hat{\mu} + \hat{\tau}_i + \hat{\beta} \bar{x}_{..} \quad \text{for } i = 1, \dots, a,$$

*overall mean of covariate values*

$$\bar{x}_{..} = \frac{1}{N} \sum_{i=1}^a \sum_{j=1}^{n_i} x_{ij}$$

*average age across all groups*

- ▶ We can equivalently write covariate-adjusted means as

$$\bar{Y}_{i.}^{\text{adj}} = \bar{Y}_{i.} - \hat{\beta} (\bar{x}_{i.} - \bar{x}_{..}) \quad \text{for } i = 1, \dots, a.$$

# Variances of covariate-adjusted means and their differences

$$\text{Ver} \left( \bar{Y}_{i.}^{\text{adj}} \right) = \dots$$

---

Contrast

---

Variance

---

$$\bar{Y}_{i.}^{\text{adj}}$$

$$\sigma^2 \left[ \frac{1}{n_i} + \frac{(\bar{x}_{i.} - \bar{x}_{..})^2}{S_{xx}} \right]$$

$$\bar{Y}_{i.}^{\text{adj}} - \bar{Y}_{i'.}^{\text{adj}}$$

$$\sigma^2 \left[ \frac{1}{n_i} + \frac{1}{n_{i'}} + \frac{(\bar{x}_{i.} - \bar{x}_{i'.})^2}{S_{xx}} \right]$$


---

where  $S_{xx} = \sum_{i=1}^a \sum_{j=1}^{n_{ij}} (x_{ij} - \bar{x}_{..})^2$ .

# Some (unadjusted) CIs in the ANCOVA model

Define the true or population-level covariate-adjusted means as

$$\mu_i^{\text{adj}} = \mu + \tau_i + \beta \bar{x}_{..} \quad \text{for } i = 1, \dots, a.$$

---

Target	$(1 - \alpha)100\%$ confidence interval
--------	---

---

$$\mu_i^{\text{adj}} \quad \bar{Y}_{i.}^{\text{adj}} \pm t_{N-a-1, \alpha/2} \sqrt{\text{MS}_{\text{Error}} \sqrt{\frac{1}{n_i} + \frac{(\bar{x}_{i.} - \bar{x}_{..})^2}{S_{xx}}}}$$

$$\mu_i^{\text{adj}} - \mu_{i'.}^{\text{adj}} \quad \bar{Y}_{i.}^{\text{adj}} - \bar{Y}_{i'.}^{\text{adj}} \pm t_{N-a-1, \alpha/2} \sqrt{\text{MS}_{\text{Error}} \sqrt{\frac{1}{n_i} + \frac{1}{n_{i'}} + \frac{(\bar{x}_{i.} - \bar{x}_{i'.})^2}{S_{xx}}}}$$

---

# VO<sub>2</sub> max data (cont)

*not adjusted*

```
y1. <- mean(vo2max$chg[vo2max$grp == "aerobic"])  
y2. <- mean(vo2max$chg[vo2max$grp == "running"])
```

*mean age*

```
x1. <- mean(vo2max$age[vo2max$grp == "aerobic"])  
x2. <- mean(vo2max$age[vo2max$grp == "running"])  
x.. <- mean(vo2max$age)
```

*$\hat{\beta}$*

```
bhat <- coef(lm_out)[3]
```

```
y1.adj <- y1. - bhat * (x1. - x..)  
y2.adj <- y2. - bhat * (x2. - x..)
```

*adjusted means*

```
n1 <- 6  
n2 <- 6  
alpha <- 0.05
```

```
tval <- qt(1-alpha/2, N-a-1)  
Sxx <- sum((vo2max$age - x..)^2)
```

```
se1 <- sqrt(MSE) * sqrt(1/n1 + (x1. - x..)^2 / Sxx)  
lo1 <- y1.adj - tval * se1  
up1 <- y1.adj + tval * se1
```

```
se2 <- sqrt(MSE) * sqrt(1/n2 + (x2. - x..)^2 / Sxx)  
lo2 <- y2.adj - tval * se2  
up2 <- y2.adj + tval * se2
```

```
se12 <- sqrt(MSE) * sqrt(1/n1 + 1/n2 + (x1. - x2.)^2 / Sxx)  
lo12 <- y1.adj - y2.adj - tval * se12  
up12 <- y1.adj - y2.adj + tval * se12
```

$$\bar{Y}_{i.}^{\text{adj}} - \bar{Y}_{i' .}^{\text{adj}} \pm t_{N-a-1, \alpha/2} \sqrt{\text{MS}_{\text{Error}} \sqrt{\frac{1}{n_i} + \frac{1}{n_{i'}} + \frac{(\bar{x}_{i.} - \bar{x}_{i' .})^2}{S_{xx}}}}$$

Table 5: Mean change in  $\text{VO}_2$  max in aerobic and running groups

	Unadjusted	Age-adjusted(CI)
Aerobic	7.71	5.19 (2.49,7.89)
Running	-2.77	-0.25 (-2.96,2.45)
Aerobic - Running	10.47	5.44 (1.45,9.43)



# Soybean data from Dr. Longnecker's notes

Soybean plants assigned to three greenhouse conditions: Supplemental lighting (SL), partial shading (PS), and control (C). The response was seed yield. The pre-treatment height of each plant was also recorded.

Yield	Height	TRT	Yield	Height	TRT	Yield	Height	TRT	Yield	Height	TRT	Yield	Height	TRT
12.2	45	C	12.4	52	C	11.9	42	C	11.3	35	C	11.8	40	C
12.1	48	C	13.1	60	C	12.7	61	C	12.4	50	C	11.4	33	C
12.3	48	C	12.2	51	C	12.6	56	C	13.2	65	C	12.3	51	C
16.6	63	SL	15.8	50	SL	16.5	63	SL	15.0	33	SL	15.4	38	SL
15.6	45	SL	15.8	50	SL	15.8	48	SL	16.0	50	SL	15.8	49	SL
15.0	35	SL	16.2	50	SL	16.7	62	SL	15.8	49	SL	15.9	52	SL
9.5	52	PS	9.5	54	PS	9.6	58	PS	8.8	45	PS	9.5	57	PS
9.8	62	PS	9.1	52	PS	10.3	67	PS	9.5	55	PS	8.5	40	PS
8.6	41	PS	10.4	67	PS	9.4	55	PS	10.2	66	PS	9.3	56	PS

Do the greenhouse conditions effect the seed yield?

What is the role of plant height (proxy for plant vigor)?

```

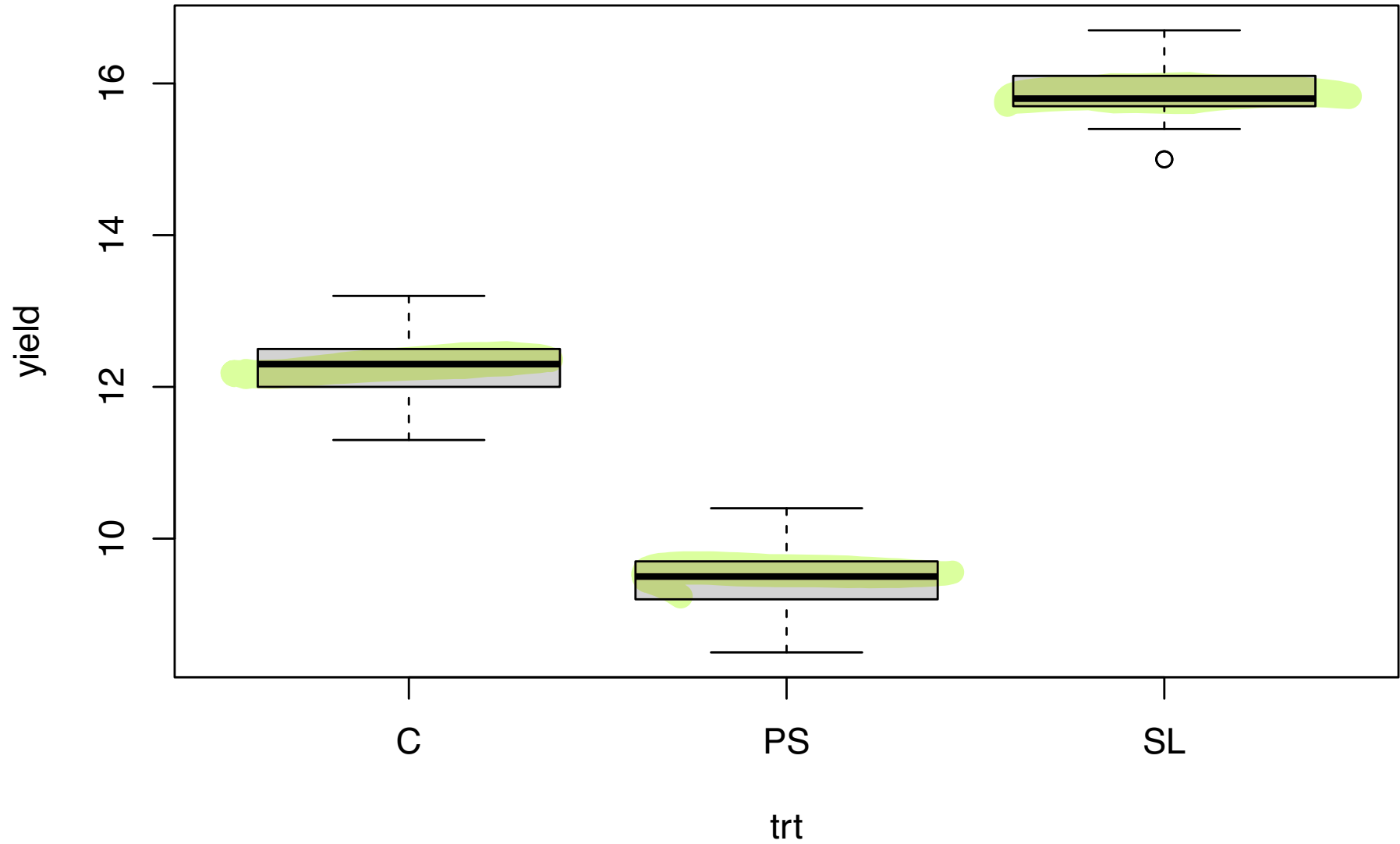
soybean <- data.frame(yield = c(12.2,12.1,12.3,16.6,15.6,15.0,9.5,9.8,8.6,
                               12.4,13.1,12.3,15.8,15.8,16.2,9.5,9.1,10.4,
                               11.9,12.7,12.6,16.5,15.8,16.7,9.6,10.3,9.4,
                               11.3,12.4,13.2,15.0,16.0,15.8,8.8,9.5,10.2,
                               11.8,11.4,12.3,15.4,15.8,15.9,9.5,8.5,9.3),
                      trt = as.factor(rep(c(rep("C",3),rep("SL",3),rep("PS",3)),5)),
                      height = c(45,48,48,63,45,35,52,62,41,
                                 52,60,51,50,50,50,54,52,67,
                                 42,61,56,63,48,62,58,67,55,
                                 35,50,65,33,50,49,45,55,66,
                                 40,33,51,38,49,52,57,40,56))

head(soybean,n = 12)

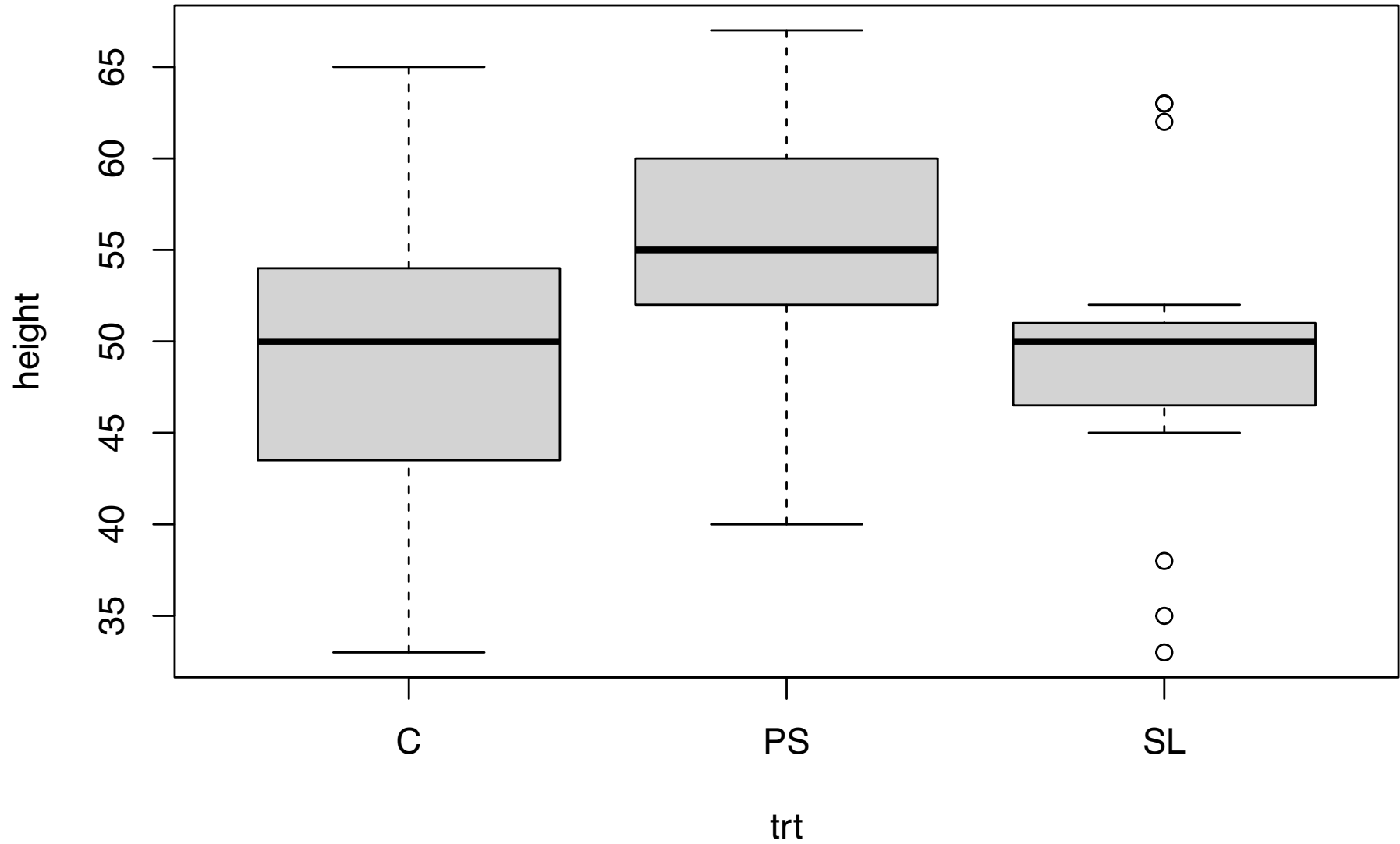
```

	yield	trt	height
1	12.2	C	45
2	12.1	C	48
3	12.3	C	48
4	16.6	SL	63
5	15.6	SL	45
6	15.0	SL	35
7	9.5	PS	52
8	9.8	PS	62
9	8.6	PS	41
10	12.4	C	52
11	13.1	C	60
12	12.3	C	51

```
boxplot(yield~trt, data = soybean)
```



```
boxplot(height~trt, data = soybean)
```





# ANCOVA allowing different slopes in each group

$$Y_{ij} = \mu + \tau_i + \beta x_{ij} + \varepsilon_{ij}$$

Assume

$$Y_{ij} = \underbrace{\mu + \tau_i}_{\mu_i} + \underbrace{(\beta + (\tau\beta)_i)}_{\beta_i} x_{ij} + \varepsilon_{ij}$$

for  $i = 1, \dots, a$ ,  $j = 1, \dots, n_i$ , where

- ▶  $Y_{ij}$  is the response of EU  $j$  in treatment group  $i$ .
- ▶  $\mu$  is a baseline or overall mean.
- ▶ the  $\tau_i$  are treatment effects.
- ▶ the  $x_{ij}$  are covariate values measured on the EUs.
- ▶  $\beta$  is a slope coefficient expressing the effect of the covariate.
- ▶ the  $(\tau\beta)_i$  allow interaction between the treatment and the covariate.
- ▶ the  $\varepsilon_{ij}$  are independent  $\text{Normal}(0, \sigma_\varepsilon^2)$  error terms.
- ▶  $n_1 + \dots + n_a = N$ . Unbalancedness not an issue.

Set  $\mu_i = \mu + \tau_i$  and  $\beta_i = \beta + (\tau\beta)_i$  for  $i = 1, \dots, a$ .

# Parameter constraints in multiple slopes model

- ▶ There are  $a$  treatment groups and  $a$  covariate slopes.
- ▶ The model has  $2(a + 1)$  parameters, which is two too-many.
- ▶ R will set  $\tau_1 = 0$  and  $(\tau\beta)_1 = 0$  to make all parameters estimable.

```
lm_out <- lm(yield ~ trt + height + trt:height, data = soybean)
summary(lm_out)
```

factor      numeric      lets treatment groups have different slopes.

Call:

```
lm(formula = yield ~ trt + height + trt:height, data = soybean)
```

Residuals:

```
      Min       1Q   Median       3Q      Max
-0.234733 -0.088745 -0.003954  0.057644  0.293320
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	9.500573	0.181934	52.220	< 2e-16 ***
trtPS	-3.639588	0.285527	-12.747	1.74e-15 ***
trtSL	3.713050	0.258575	14.360	< 2e-16 ***
height	0.056298	0.003644	15.451	< 2e-16 ***
trtPS:height	0.009102	0.005372	1.694	0.0982 .
trtSL:height	-0.002437	0.005179	-0.470	0.6407

---  
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.1256 on 39 degrees of freedom  
Multiple R-squared: 0.9981, Adjusted R-squared: 0.9978  
F-statistic: 4054 on 5 and 39 DF, p-value: < 2.2e-16

$\hat{\mu}$        $\hat{\sigma}_{PS}$        $\hat{\sigma}_{SL} = 0$

$\hat{\sigma}_{SL}$

$\hat{\beta}$

$(\hat{\sigma}_{\beta})_{PS}$

$(\hat{\sigma}_{\beta})_{SL}$



```
parms <- coef(lm out)  
parms
```

```
(Intercept)      trtPS      trtSL      height trtPS:height trtSL:height  
9.500572519 -3.639588070  3.713050304  0.056297710  0.009101605 -0.002436573
```

---



# An F-test for equal slopes

Define the error sums of squares:

SS	Formula
$SS_{\text{Error}}(\text{Full})$	$\sum_{i=1}^a \sum_{j=1}^{n_{ij}} (Y_{ij} - (\hat{\mu} + \hat{\tau}_i + (\hat{\beta} + (\hat{\tau}\beta)_i)x_{ij}))^2$
$SS_{\text{Error}}(\text{Equal slopes})$	$\sum_{i=1}^a \sum_{j=1}^{n_{ij}} (Y_{ij} - (\hat{\mu} + \hat{\tau}_i + \hat{\beta}x_{ij}))^2$

different intercept
single, common slope

Now set

$$F_{T \times C} = \frac{[SS_{\text{Error}}(\text{Equal slopes}) - SS_{\text{Error}}(\text{Full})]/(a - 1)}{SS_{\text{Error}}(\text{Full})/(N - 2a)}$$

Reject  $H_0$ : *Equal slopes* at  $\alpha$  if  $F_{T \times C} > F_{a-1, \cancel{N-2a}, \alpha}$ .

$N-2a$

# Soybean data (cont)

```
lm_eqslp <- lm(yield ~ trt + height, data = soybean) # equal slopes model  
lm_full <- lm(yield ~ trt + height + trt:height, data = soybean) # full model
```

```
SSE_eqslp <- sum(lm_eqslp$resid^2)  
SSE_full <- sum(lm_full$resid^2)
```

```
a <- 3 (ps, SL, C)
```

```
N <- nrow(soybean)
```

$\uparrow n_1 + n_2 + n_3$

```
FTrtCov <- ((SSE_eqslp - SSE_full) / (a-1)) / (SSE_full / (N - 2*a))
```

```
pTrtCov <- 1 - pf(FTrtCov, a-1, N - 2*a)
```

*(omit interaction)*

Same as interaction p value from `anova()` on the `lm()` output.

```
anova(lm_out)
```

Analysis of Variance Table

Response: yield

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
<del>trt</del>	<del>2</del>	<del>308.134</del>	<del>154.067</del>	<del>9770.7982</del>	<del>&lt; 2e-16 ***</del>
<del>height</del>	<del>1</del>	<del>11.389</del>	<del>11.389</del>	<del>722.2854</del>	<del>&lt; 2e-16 ***</del>
trt:height	2	0.079	0.039	2.4938	0.09568 .
Residuals	39	0.615	0.016		

---  
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

p-value for testing  
 $H_0: \beta_1 = \dots = \beta_a$

# When slopes are unequal

If we reject  $H_0$ : *Equal slopes*, then

- ▶ We can compute covariate-adjusted means with the different slopes.
- ▶ We can compare the covariate adjusted means—but the CI formulas are different from the ones above (details omitted).

# References

Kuehl, R. O. 2000. *Design of Experiments: Statistical Principles of Research Design and Analysis*. Duxbury/Thomson Learning.