

STAT 712 fa 2021 Exam 2

1. Let $X_1 \sim \text{Exponential}(\beta_1)$ and $X_2 \sim \text{Exponential}(\beta_2)$ be independent random variables and consider $R = X_1/X_2$ and $U = X_2$.
 - (a) Give the joint pdf of random variable pair (R, U) .
 - (b) State whether R and U are independent and explain how you know.
 - (c) Give the marginal pdf of R .
 - (d) Give the conditional pdf of $U|R = r$.

2. Consider the pair of random variables (X, Y) arising from the hierarchical model

$$Y|X \sim \text{Normal}(X, 1)$$

$$X \sim p_X(x) = (1/2) \cdot \mathbf{1}(x = 1) + (1/2) \cdot \mathbf{1}(x = -1).$$

- (a) Give the mean and variance of Y .
- (b) Give the marginal pdf of Y .
- (c) Give the covariance of X and Y .
- (d) Give the correlation of X and Y .

3. Let $Z_1, \dots, Z_8 \stackrel{\text{ind}}{\sim} \text{Normal}(0, 1)$. Use Z_1, \dots, Z_8 to construct a random variable having the
- (a) $\text{Normal}(0, 1/8)$ distribution.
 - (b) χ_4^2 distribution.
 - (c) t_5 distribution.
 - (d) $F_{4,4}$ distribution.

4. Let X_1, X_2, \dots be independent random variables such that

$$\log(\mathbb{E} \exp(tX_i)) = -(1/2)^i \log(1-t) \quad \text{for } t < 1$$

for $i = 1, 2, \dots$. Now, let $Y_n = \sum_{i=1}^n X_i$ for $n \geq 1$.

- (a) Give the moment generating function $M_{Y_n}(t)$ of Y_n .
- (b) Give the limit as $n \rightarrow \infty$ of $M_{Y_n}(t)$ and identify the distribution to which it belongs.