STAT 712 fa 2022 Exam 1

1. Let A, B, and C be events such that A and B are independent with $P(A) = 1/2$ and $P(B) = 1/3$, and

$$P(C|A^c \cap B) = P(C|A \cap B^c) = P(C|A \cap B) = P(C^c|A^c \cap B^c) = 3/4.$$

   (a) Give $P(C \cap A^c \cap B^c)$.
   (b) Give $P(C \cap A^c)$.
   (c) Give $P(A|C)$.

2. Let $X \sim f_X(x) = \alpha e^{\alpha x} e^{-e^{\alpha x}}$ for all $x \in \mathbb{R}$ for some $\alpha > 0$. Let $Y = e^{\alpha X}$.

    (a) Give the pdf of Y. Make sure to define it for all $y \in \mathbb{R}$.

    (b) Give the mgf of Y.

    (c) Give $\mathbb{E}(Y - \mathbb{E}Y)^3$.

3. Consider the pdf given by

$$f(x) = \begin{cases} 0, & x < 0 \\ x, & 0 \le x < 1 \\ 2 - x, & 1 \le x < 2 \\ 0, & 2 \le x. \end{cases}$$

(a) For $X \sim f$, give $\mathbb{E}X$.

(b) Give the cdf F corresponding to the pdf f. Make sure to define it for all $x \in \mathbb{R}$.

(c) Suppose $U \sim \text{Uniform}(0, 1)$. Explain how you would find a transformation g such that $X = g(U)$ has pdf f (you do not need to give the transformation).

4. Let $U \sim \text{Uniform}(0, 1)$ and let $V = 1 - U$. Show that U and V are identically distributed (have the same cdf).