

STAT 712 fa 2022 Exam 2

1. Let $T|U \sim \text{Normal}(cU, 1)$ and $U \sim \text{Normal}(0, 1)$ for some $c \in \mathbb{R}$.
 - (a) Give $\mathbb{E}T$ and $\text{Var} T$.
 - (b) Give $\text{Corr}(T, U)$.
 - (c) Derive the marginal pdf of T .

2. Let (X_1, X_2) have joint pdf given by

$$f(x_1, x_2) = \frac{1}{x_2^\lambda} \exp\left[-\frac{x_1}{x_2^\lambda}\right] \mathbf{1}(0 < x_2 < 1, x_1 > 0)$$

for some $\lambda > 0$.

- (a)
 - i. Give the marginal pdf of X_2 .
 - ii. Give the conditional pdf of X_1 given $X_2 = x_2$.
 - iii. Give $\mathbb{E}X_1$ and $\text{Var } X_1$.
- (b)
 - i. Find the joint pdf of $R = X_1/X_2$ and $V = X_2$.
 - ii. State whether R and V are independent. Explain.
 - iii. Find the marginal pdf of R .

3. Let Z_1, Z_2, Z_3, Z_4 be independent $\text{Normal}(0, 1)$ rvs.
- (a) Give the distribution of each of the following:
- $(Z_1 + Z_2)/(\sqrt{2}|Z_3|)$
 - $(1/6)(Z_1 - 2Z_2 + Z_3)^2$
 - $Z_1^2/(Z_2/\sqrt{2} - Z_3/\sqrt{2})^2$
 - $(1/2)[(Z_1 - Z_2)^2 + (Z_1 + Z_2)^2]$
 - $(1/3)[(Z_1 - Z_2)^2 + (Z_2 - Z_3)^2 + (Z_1 - Z_3)^2]$
- (b) Give
- $\text{Cov}(Z_1 + Z_2, Z_1 - Z_2)$
 - $\text{Cov}(Z_1 + Z_2 + Z_3 + Z_4, Z_1 - Z_2 + Z_3 - Z_4)$