STAT 712 hw 1

Set theory, probability axioms, counting

Do problems 1.2, 1.3, 1.8, 1.10, 1.11, 1.12, 1.13, 1.14, 1.18, 1.19, 1.23 from CB. In addition:

1. For a collection of sets $A_1, \ldots, A_n$, $n \geq 2$, we have

$$P(\cup_{i=1}^n A_i) = \sum_{i=1}^n P(A_i) - \sum\sum_{1 \leq i_1 < i_2 \leq n} P(A_{i_1} \cap A_{i_2}) + \sum\sum\sum_{1 \leq i_1 < i_2 < i_3 \leq n} P(A_{i_1} \cap A_{i_2} \cap A_{i_3}) - \cdots + (-1)^{n+1} P(\cap_{i=1}^n A_i).$$

This is known as the *inclusion-exclusion* principle.

(a) Prove this result by induction. That is, prove it for $n = 2$ and then show that if it is true for an arbitrary $n \geq 2$, it must be true for $n + 1$.

(b) Suppose n guests take n seats around a table at random. Then the host rearranges them according to a seating chart he made before the guests' arrival.

i. Find the probability that every guest must move to a different seat. *Hint: Let A_i be the event that guest i sits in his or her assigned seat for $i = 1, \ldots, n$.*

ii. Find the limit of this probability as $n \to \infty$.

(c) Two 52-card decks are shuffled and place side-by-side. From each deck a card is drawn and placed face-up. This is repeated 52 times, resulting in 52 pairs of cards drawn. What is the probability that at least one pair is a match?