STAT 712 hw 3

Expected value, variance, mgfs

Do problems 2.14, 2.17, 2.24, 2.26, 2.28, 2.32, 2.38 from CB. In addition:

1. Each of N visitors entering a museum must pass through one of n turnstiles. Suppose each visitor chooses a turnstile at random, independently of the others, and let X_n be the number of visitors that enter through turnstile n.
   (a) Give the pmf of X_n.
   (b) Give the mgf M_{X_n} of X_n.
   (c) For a positive integer k, let $N = k \cdot n$, so that k visitors per turnstile enter, and find $\lim_{n\to\infty} M_{X_n}$.
   (d) Give the limiting distribution of X_n as $n \to \infty$ when $N = k \cdot n$.
2. (Optional) A rv X is called b-sub-Gaussian if for some $b > 0$, $\mathbb{E}e^{tX} \leq e^{b^2t^2/2}$ for all $t \in \mathbb{R}$.
   (a) Show that if X is b-sub-Gaussian, then
   $$P(|X| \geq a) \leq 2e^{-a^2/(2b^2)} \quad \text{for all } a > 0.$$
   (b) Show that $X \sim \text{Normal}(0, \sigma^2)$ is a σ-sub-Gaussian rv.
   (c) Show that $X \sim \text{Uniform}(-b, b)$ is a b-sub-Gaussian rv.
   Hint: Write the mgf of X as an infinite series and use the inequality $(2k + 1)! \geq k!2^k$.