STAT 713 hw 3

Bayesian estimators, MLEs, MoMs, bias and mean squared error

Do problems 7.19, 7.23, 7.50 from CB. In addition:

1. Suppose $X_1, \ldots, X_n \overset{\text{ind}}{\sim} f_X(x; \alpha, \beta) = \beta\alpha^\beta x^{-(\beta+1)}\mathbf{1}(x > \alpha)$.
   (a) Give expressions for α and β in terms of the τ_1 and τ_2 quantiles ξ_{τ_1} and ξ_{τ_2}.
   (b) (Optional) Run a simulation with 10,000 datasets to obtain (an approximation of) the MSE of the quantile estimators of α and β corresponding to your work in part (a) under $\tau_1 = 0.1$ and $\tau_2 = 0.9$ when $\alpha = 1$, $\beta = 2$, and $n = 50$.

2. Let $X_1, \ldots, X_n \overset{\text{ind}}{\sim} f(x; \theta) = \theta x^{\theta-1}\mathbf{1}(0 < x < 1)$ for $\theta > 0$.
   (a) Find the method of moments estimator of θ.
   (b) Use Jensen's inequality to show that this estimator is biased.

3. Let $X_1, \ldots, X_n \overset{\text{ind}}{\sim} \text{Gamma}(\alpha_0, \beta)$, $\beta > 0$ with α_0 known.
   (a) Find the MLE $\hat{\tau}$ of $\tau = 1/\beta$.
   (b) Find the constant c such that $c\hat{\tau}$ is unbiased for τ.
   (c) Find the constant c that minimizes the mean squared error of $c\hat{\tau}$.

4. (Optional) Consider the Bayesian hierarchical model

$$X_1, \ldots, X_n|\theta \overset{\text{ind}}{\sim} \text{Normal}(\theta, \sigma^2)$$
$$\theta \sim \pi(\theta) = \exp(-|\theta|/\lambda)/(2\lambda),$$

   for some known constants $\lambda > 0$ and $\sigma > 0$. Find the posterior mode of $\theta|X_1, \ldots, X_n$.