STAT 713 hw 8

Asymptotic tests and interval estimators

Do problems 9.3, 9.4, 9.12, 9.13, 9.17 from CB. In addition:

1. Let $X_1, \ldots, X_n \overset{\text{ind}}{\sim} \text{Gamma}(2, \beta)$, $\beta > 0$.
   (a) Show that the LRT for H_0: $\beta = \beta_0$ has a rejection rule of the form $\hat{\beta}_n/\beta_0 < c_1$ or $\hat{\beta}_n/\beta_0 > c_2$, where $\hat{\beta}_n$ is the MLE and c_1 and c_2 satisfy $c_1 < c_2$ and $c_1 e^{-c_1} = c_2 e^{-c_2}$.
   (b) For $n = 10$, find the values of c_1 and c_2 under which the LRT has size 0.05. You will need to search for these values numerically.
   (c) For $n = 10$, compare c_1 and c_2 to the 0.025 and 0.975 quantiles of the distribution of $\hat{\beta}_n/\beta_0$ under H_0: $\beta = \beta_0$. *These "equal tails" critical values are used more commonly in practice than c_1 and c_2 and are much easier to find!*
   (d) Give the form of the CI for β based on inverting the size-α LRT test of H_0: $\beta = \beta_0$.
   (e) Give the form of the CI for β based on inverting the size-α score test of H_0: $\beta = \beta_0$.
   (f) Give the form of the CI for β obtained by inverting the cdf of $\sum_{i=1}^n X_i$.
   (g) Justify the confidence interval $\hat{\beta}_n \pm z_{\alpha/2}\hat{\beta}_n/\sqrt{2n}$. What name would you give it?
   (h) Construct the 95% confidence intervals from parts (d), (e), (f), and (g) using the data

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X <- c(0.99, 10.63, 7.70, 5.23, 4.20, 10.74, 2.69, 7.37, 4.51, 9.05)
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   (i) Set $\beta = 3$, $n = 10$, and generate 5000 datasets; on each data set record for each of the four intervals i) whether it contained the true value of β and ii) its width. Report the proportion of times each interval contained its target and its average width.