STAT 714 hw 4

Projections, generalized inverses, least-squares, model reparameterization

Do problems 2.8 and 2.11 from Monahan. In addition:

1. Let $\mathbf{P}$ be the matrix of an orthogonal projection onto a subspace W of $\mathbb{R}^n$.
   (a) Show that $\mathbf{P}$ is symmetric.
   (b) Show that for any vector $\mathbf{v} \in \mathbb{R}^n$ we have $\|\mathbf{P}\mathbf{v}\| \leq \|\mathbf{v}\|$.
2. Let $\mathbf{A}$ be an $m \times n$ matrix with rank r and reduced singular value decomposition $\mathbf{A} = \mathbf{U}_r\mathbf{D}\mathbf{V}_r^T$. Show that $\mathbf{V}_r\mathbf{D}^{-1}\mathbf{U}_r^T$ is a generalized inverse of $\mathbf{A}$.
3. For a symmetric projection matrix, show that the trace is equal to the rank.
4. For any $n \times p$ matrix $\mathbf{X}$:
   (a) Let $\mathbf{W} = (\mathbf{I}_n - n^{-1}\mathbf{1}_n\mathbf{1}_n^T)\mathbf{X}$ and check whether the model $\mathbf{y} = \mathbf{W}\mathbf{d} + \mathbf{e}$ is a reparameterization of the model $\mathbf{y} = \mathbf{X}\mathbf{b} + \mathbf{e}$ for any design matrix $\mathbf{X}$. If it is not, give an example of a design matrix $\mathbf{X}$ for which it is not true.
   (b) Let $\tilde{\mathbf{X}} = [\mathbf{1}_n \;\; \mathbf{X}]$ and $\tilde{\mathbf{W}} = [\mathbf{1}_n \;\; (\mathbf{I}_n - n^{-1}\mathbf{1}_n\mathbf{1}_n^T)\mathbf{X}]$ and check whether the model $\mathbf{y} = \tilde{\mathbf{W}}\mathbf{d} + \mathbf{e}$ is a reparameterization of the model $\mathbf{y} = \tilde{\mathbf{X}}\mathbf{b} + \mathbf{e}$ for any design matrix $\mathbf{X}$. If it is not, give an example of a design matrix $\mathbf{X}$ for which it is not true.
5. Let $\mathbf{X}$ be an $n \times p$ matrix. Give a matrix $\mathbf{P}$ which is not the identity matrix such that $\mathbf{P}\mathbf{c} = \mathbf{c}$ for any $\mathbf{c} \in \operatorname{Col} \mathbf{X}^T$.