STAT 515 – Section 8.6 Supplement

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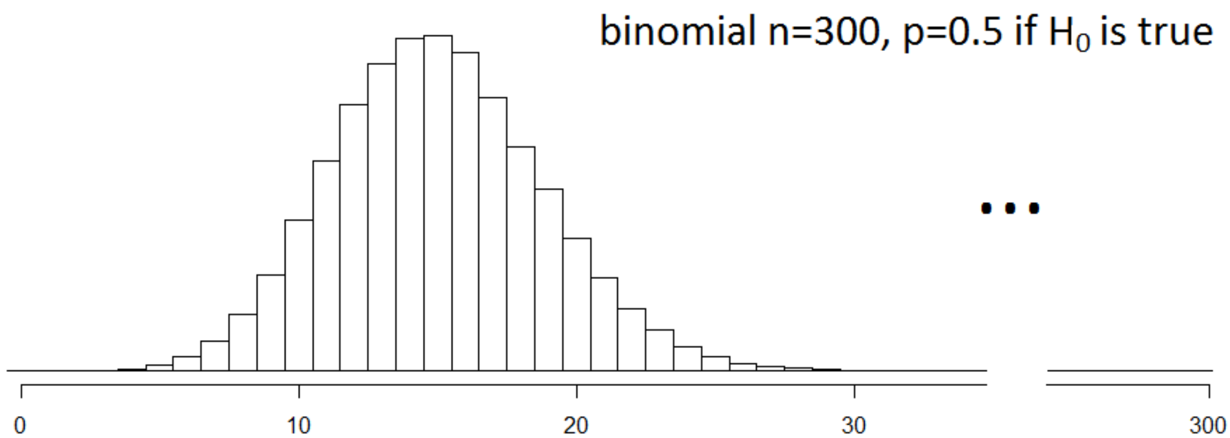
Examples 8.10 and 8.11 in Section 8.6 consider what percent of all batteries from a manufacturer are defective. In particualr, 300 batteries are randomly selected from a very large shipment to test H_0: p=0.05 vs. H_A: p<0.05 at the a=0.01 level. Ten of the 300 in the sample are found to be defective.

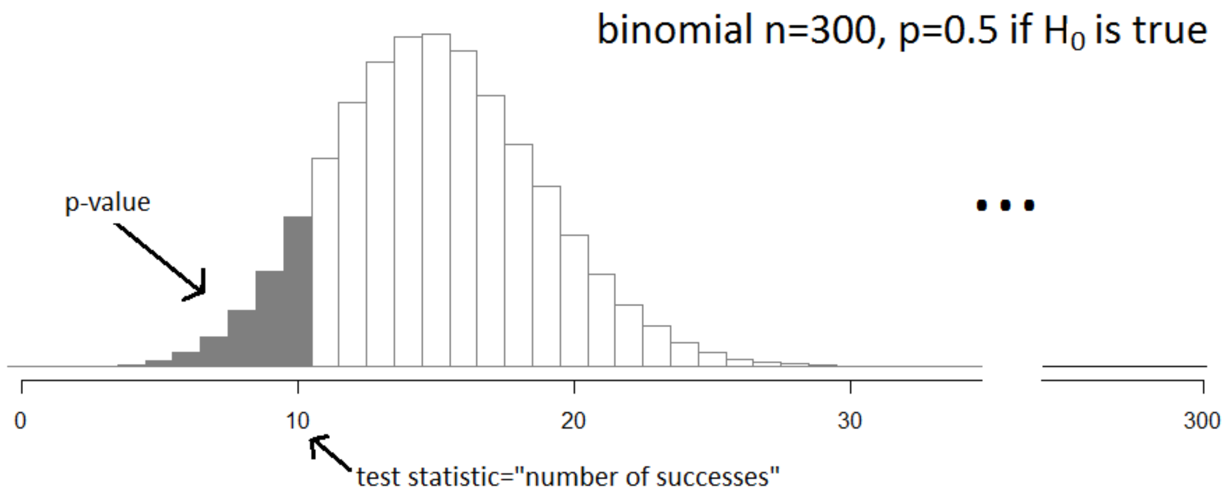
Since the population the 300 batteries are selected from is much larger, this is approximately a binomial experiment. The book chooses to tackle this problem using the central limit theorem (without even using the normal approximation to the binomial!). Since we have computers that can calculate binomial probabilities very accurately, there is no reason to use an approximation.

If H_0 is true for this problem, then the number of statistics observed should follow a binomial distribution with n=300 and p=0.05.



The p-value is the probability of observing a test statistic at least as extreme as the test statistic (we observed 10) if the null hypothesis is true. If the alternate was H_A: p<0.0.5, then extreme is the true p being smaller, which means the observed value should tend to be small. That means the p-value would be the probability that the binomial was less than or equal to 10. Similarly, if the alternate was H_A: p>0.05, then the p-value would be the probability that the binomial was greater than or equal to 10. For the alternate H_A: p≠0.05, the p-value is the smaller of those two values.

In this case we are testing H_A: p<0.05, so we want the probability the binomial is less than or equal to 10.



We can find this on R using the command `pbinom`,

```
> hist(d,breaks=(0:301)-0.5)
> pbinom(10,300,.05)
[1] 0.1123014
```

or the function `binom.test`,

```
> binom.test(10,300,.05,alternative="less")

        Exact binomial test

data:  10 and 300
number of successes = 10, number of trials = 300, p-value = 0.1123
alternative hypothesis: true probability of success is less than 0.05
95 percent confidence interval:
 0.00000000 0.05588474
sample estimates:
probability of success
            0.03333333
```

With a p-value of 0.1123 > α=0.01 we fail to reject the null hypothesis. We do not have significant evidence that $p<0.05$.

The only assumption required to do this is that we have a binomial distribution (there is no sample size requirement).

This is the same conclusion the text reached in example 8.11 where they found a p-value of 0.093. However at α=0.10 we would have come to different conclusions! In this case, using the central limit theorem with no continuity correction like the book did would cause you to think you had more evidence than you actually did. Using the continuity correction makes it much closer (a p-value of 0.1166 if you work it out), but there is no reason to do so.