Practice Probability Problems:

Recall: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $P(A \cap B) = P(A)P(B|A)$

1) Consider an experiment where 2 balls are drawn from a bin containing 3 red balls and 2 green balls (the balls are not replaced between draws). Define the events A, B, and C as follows:
A={Both balls are red}, B={Both balls are green}, C={The first ball is red}

a) What is the sample space for this experiment
b) What is the probability associated with each of the sample points
c) Find the probability of A, B, and C
d) Find $P(A|B)$ and $P(B|A)$.
e) Are the events A and B independent, mutually exclusive, or dependent in some other way?
f) Find $P(A|C)$ and $P(C|A)$
g) Are the events A and C independent, mutually exclusive, or dependent in some other way?

2) Consider a fair 6 sided dice where one side is labeled 1, one side is labeled 5, and the others are all labeled 0. The die is rolled twice. Define the events A, B, and C as follows:
A={The first die is a 1}, B={The first die is a 5}, C={The second die is a 0}

a) What is the sample space for this experiment
b) What is the probability associated with each of the sample points
c) Find the probability of A, B, and C
d) Find $P(A \cup B)$ and $P(A \cap B)$
e) Are the events A and B independent, mutually exclusive, or dependent in some other way?
f) Find $P(A \cup C)$ and $P(A \cap C)$
g) Are the events A and C independent, mutually exclusive, or dependent in some other way?

3) Consider the experiment in question 2. Let X=the sum of the two numbers.

a) Find the probability distribution of X
b) Find the mean and standard deviation of X
c) If the number was the amount of $ won in a dice game, how much is each chance at the game worth?

The answers and one possible way of getting them... your reasoning can be different, just make sure you can understand this reasoning:

1a) RR, RG, GR, GG
b) P(RR) = P(red 1^{st}) x P(red 2^{nd} | red 1^{st}) = 3/5 x 2/4 = 6/20 = 0.3
P(RG) = 3/5 x 2/4 = 6/20 = 0.3 P(GR) = 2/5 x 3/4 = 6/20 = 0.3 P(GG) = 2/5 x 1/4 = 2/20 = 0.1
c) P(A)=P(RR)=0.3 P(B)=P(GG)=0.1 P(C)=P(RG)+P(RR)=0.3+0.3=0.6
d) P(A|B) = P(A∩B)/P(B) = P(Both balls are red AND Both balls are green)/P(B) = 0/0.1=0
because there are no sample points that are in both A and B... same for P(B|A)
e) Mutually exclusive
f) P(A|C) = P(A∩C)/P(C) = P(Both balls are red AND First ball is red)/P(C) = P(RR)/P(C)=0.3/0.6 = 0.5
P(C|A) = P(C∩A)/P(A) = P(First ball is red AND Both balls are red)/P(A) = P(RR)/P(A)=0.3/0.3 = 1.0
g) A and C are dependent

2a) 00, 01, 10, 11, 05, 50, 55, 15,51
b) P(11)=P(55)=P(15)=P(51) = 1/6 x 1/6 = 1/36
P(00) = 4/6 x 4/6 = 16/36=4/9
P(01)=P(10)=P(50)=P(05) = 4/6 x 1/6 = 4/36= 1/9
c) You should be able to get these three without doing all this work (you are looking at one die at a time)
P(A) = P(10)+P(11)+P(15) = 4/36+1/36+1/36 = 6/36 = 1/6
P(B) = P(50)+P(51)+P(55) = 4/36+1/36+1/36 = 6/36 = 1/6
P(C) = P(10)+P(50)+P(00) = 4/36 + 4/36 + 16/36 = 24/36 = 4/6 = 2/3
d) P(A∩B) = 0 because these two events have no sample points in common
and so P(AUB)=P(A)+P(B)=1/6+1/6=2/6=1/3
e) They are mutually exclusive
f) P(A∩C) = P(1^{st} die is a one AND 2^{nd} die is a zero) = P(10)=4/36 = 1/9
P(AUC) = P(A) + P(C) - P(A∩C) = 6/36 + 24/36 - 4/36 = 26/36 = 13/18
g) Independent because P(A|C) = P(A∩C)/P(C)= (1/9) / (2/3) = 3/18 = 1/6 = P(A)
and P(C|A) = P(A∩C)/P(A)= (1/9) / (1/6) = 6/9 =2/3 = P(C)

3a) Figure out which sample points go with which sum and add up the probabilities

x	0	1	2	5	6	10
P(X=x)	4/9	2/9	1/36	2/9	1/18	1/36

b) $\mu = \Sigma xp(x) = 0x4/9 + 1x2/9 + 2x1/36 + 5x2/9 + 6x1/18 + 10x1/36$
$= 0 + 2/9 +2/36 + 10/9 + 6/18 + 10/36 = 72/36 = 2$
$\sigma^2 = \Sigma(x-\mu)^2p(x) = (0-2)^2(4/9) + (1-2)^2(2/9) + (2-2)^2(1/36) + (5-2)^2(2/9) + (6-2)^2(1/18) + (10-2)^2(1/36)$
$= 16/9 + 2/9 + 0 + 18/9 + 16/18 + 64/36 = 240/36 \approx 6.67$

c) The expected value = μ = about \$6.67.