Today

- Functions of Continuous Distributions (continued)

- Joint Distributions

2.2.2-The Gamma Distribution

\[ g(t) = \frac{\lambda^\alpha}{\Gamma(\alpha)} t^{\alpha-1} e^{-\lambda t} \quad \text{for } t \geq 0 \]

2.2.3 – The Normal Distribution

\[ f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad \text{for } -\infty < x < \infty \]
2.3 - Functions of Random Variables

Let $Y = aX + b$

$F_Y(y) = F_X((y - b)/a)$

$f_Y(y) = (1/a) f_X((y - b)/a)$

e.g. if $X \sim N(\mu, \sigma^2)$ then $(X - \mu)/\sigma \sim \mathcal{Z}$

$Y = g(X):$ Let $X$ be a continuous RV with p.d.f. $f(x)$ and $Y = g(X)$, where $g$ is differentiable and strictly monotone everywhere that $f(x) > 0$.

Then

$$f_Y(y) = f_X(g^{-1}(y)) \left| \frac{d}{dy} g^{-1}(y) \right|$$

Page 60: “For any specific problem, it is usually easier to proceed from scratch than to decipher the notation and apply the proposition.”
Example) Let $X=Z^2$ where $Z \sim \mathcal{N}(0,1)$.

Example 2) Let $X=F^{-1}(U)$ where $U$ is uniform on $[0,1]$ and $F$ is a CDF.

Chapter 3 – Joint Distributions

The joint behavior of two random variables $X$ and $Y$ is determined by their CDF:

$$F_{XY}(x,y) = P(X \leq x, Y \leq y)$$
We can use this definition to find the area of any given rectangle:

\[ P(x_1 < X \leq x_2, y_1 < Y \leq y_2) = F_{XY}(x_2, y_2) - F_{XY}(x_1, y_2) - F_{XY}(x_2, y_1) + F_{XY}(x_1, y_1), \]
for \( x_1 < x_2, y_1 < y_2 \).

3.2 - Discrete R.V.'s

For discrete R.V.'s the joint p.m.f. is

\[ p(x, y) = P(X=x, Y=y) \]

Example) A fair coin is tossed three times. Let \( X = \) number of heads in three tossings and \( Y = \) difference (in absolute values) between the number of heads and number of tails.