STAT 702/J702  
October 5th, 2004  
- Lecture 14 -

Instructor: Brian Habing  
Department of Statistics  
Telephone: 803-777-3578  
E-mail: habing@stat.sc.edu

Today

- Homework
- Joint Distributions (continued)

Chapter 2 #67: The Weibull has CDF

\[ F(x) = 1 - e^{-\left(\frac{x}{\alpha}\right)^\beta} \]

for \( x \geq 0, \alpha > 0, \text{ and } \beta > 0 \)

a) Find the density function.
b) Show if $W$ follows a Weibull, then $X=\left(\frac{W}{\alpha}\right)^\beta$ follows an exponential.

c) How could Weibull random variables be generated from a uniform random generator?

Also) Use R to plot the pdf for a few values of alpha and beta to demonstrate how they affect the behavior of the Weibull distribution.
Chapter 3 – Joint Distributions

The joint behavior of two random variables $X$ and $Y$ is determined by their CDF:

$$F_{XY}(x,y) = P(X \leq x, Y \leq y)$$

3.2 - Discrete R.V.’s

For discrete R.V.’s the joint p.m.f. is

$$p(x,y) = P(X=x, Y=y)$$

Example) A fair coin is tossed three times. Let $X$=number of heads in three tossings and $Y$= difference (in absolute values) between the number of heads and number of tails.
The Marginal p.m.f of $X$ is
$$p_X(x) = \sum_y p(x, y)$$

The Conditional p.m.f. of $X$ is
$$p_{X|Y}(x | y) = P(X=x | Y=y) = \frac{P(X=x, Y=y)}{P(Y=y)} = \frac{p_{XY}(x, y)}{p_Y(y)}$$

$X$ and $Y$ are independent if
$$F_{XY}(x, y) = F_X(x) F_Y(y)$$

This implies that
$$p_{XY}(x, y) = p_X(x) p_Y(y)$$

It also works for functions $g(x)$ and $h(y)$.

### 3.3 Continuous R.V.'s

Continuous $(X, Y)$ have joint cdf
$$F_{XY}(x,y)=P(X \leq x, Y \leq y)$$

The joint pdf is
$$f_{XY}(x, y) = \frac{\partial^2}{\partial x \partial y} F_{XY}(x, y)$$
So

\[ F(x, y) = \int_{-\infty}^{x} \int_{-\infty}^{y} f(u, v) \, du \, dv \]

and

\[ P((x, y) \in A) = \int_{A} \int f(x, y) \, dx \, dy \]

The marginal p.d.f.’s are analogous to the marginal p.m.f.’s for discrete variables, but are defined using integrals:

\[ f_x(x) = \int_{-\infty}^{\infty} f(x, y) \, dy, \quad f_y(y) = \int_{-\infty}^{\infty} f(x, y) \, dx. \]

The conditional p.d.f.’s are also analogous:

\[ f_{Y\mid X}(y \mid x) = \frac{f_{X,Y}(x, y)}{f_X(x)} \]
Example 1) \( f(x, y) = 2, \ 0 < x < y < 1 \)
and 0 elsewhere

Example 2)

\[
f(x, y) = \lambda^2 \exp(-y\lambda) \\
0 \leq x \leq y \leq 1, \ \lambda > 0
\]

Example 3)

\[
f(x, y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \exp \left\{ -\frac{1}{2(1-\rho^2)} \left[ \frac{(x-\mu_x)^2}{\sigma_x^2} + \frac{(y-\mu_y)^2}{\sigma_y^2} \right] - \frac{2\rho(x-\mu_x)(y-\mu_y)}{\sigma_x\sigma_y} \right\}
\]

\[-\infty < \mu_x, \mu_y < \infty, \ \sigma_x, \sigma_y > 0, \ -1 < \rho < 1\]