Today

• Homework

• Continuous Joint Distributions (continued)

Chapter 2: #67. Also use R to plot the pdf for a few values of alpha and beta to demonstrate how they affect the behavior of the Weibull distribution.
Example 2)

\[ f(x, y) = \lambda^2 \exp(-y \lambda) \]

\[ 0 \leq x \leq y \leq 1, \lambda > 0 \]

\[ F(x_0, y_0) = \int_{-\infty}^{x_0} \int_{-\infty}^{y_0} f(x, y) \, dx \, dy \]

\[ f_X(x) = \int_{-\infty}^{\infty} f(x, y) \, dy \]

\[ f_Y(y) = \int_{-\infty}^{\infty} f(x, y) \, dx \]

\[ f_{Y|X}(y \mid x) = \frac{f_{X,Y}(x, y)}{f_X(x)} \]

\[ f_{X|Y}(x \mid y) = \frac{f_{X,Y}(x, y)}{f_Y(y)} \]
Example 3)

\[
 f(x, y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \exp \left\{ -\frac{1}{2(1-\rho^2)} \left[ \frac{(x - \mu_x)^2}{\sigma_x^2} + \frac{(y - \mu_y)^2}{\sigma_y^2} - \frac{2\rho(x - \mu_x)(y - \mu_y)}{\sigma_x\sigma_y} \right] \right\},
\]

\(-\infty < \mu_x, \mu_y < \infty, \quad \sigma_x, \sigma_y > 0, \quad -1 < \rho < 1\)

\[
 f_X(x) = \int_{-\infty}^{\infty} f(x, y)dy
\]

\[
 f_Y(y) = \int_{-\infty}^{\infty} f(x, y)dx
\]

\[
 f_{Y\mid X}(y \mid x) = \frac{f_{X\mid Y}(x, y)}{f_X(x)}
\]

\[
 f_{X\mid Y}(x \mid y) = \frac{f_{X\mid Y}(x, y)}{f_Y(y)}
\]
3.6.2 – Functions of Joint Random Variables

(X, Y) have joint pdf \( f_{XY}(x, y) \).

We want the distribution of 
\( U=g_1(X, Y), \ V=g_2(X, Y) \).

E.g.: \( U=X+Y, \ V=Y^2 \) or \( U=X+Y, \ V=X-2Y, \) etc.

For the continuous case the joint pdf of \((U,V)\) is

\[
f_{U,V}(u, v) = f_{X,Y}(h_1(u, v), h_2(u, v)) \left| J \right|
\]

where \( h_1 \) and \( h_2 \) are the inverse functions: 
\( x = h_1(u, v), \ y = h_2(u, v) \)

And \( J \) is the Jacobian

\[
J = \begin{vmatrix}
\frac{dh_1}{du} & \frac{dh_1}{dv} \\
\frac{dh_2}{du} & \frac{dh_2}{dv}
\end{vmatrix}
\]

Example 1) \( X \) and \( Y \) have joint p.d.f.

\[
f_{XY}(x, y) = 2 \quad 0 \leq x, y \leq 1
\]

\( U=X/Y \) and \( V=Y \)

Find the joint and marginal p.d.f's of \( X \) and \( Y \).