



$$f_X(x) = \int_{-\infty}^{\infty} f(x, y) dy$$
$$f_Y(y) = \int_{-\infty}^{\infty} f(x, y) dx$$

$$f_{Y|X}(y \mid x) = \frac{f_{X,Y}(x, y)}{f_X(x)}$$

$$f_{X|Y}(x \mid y) = \frac{f_{X,Y}(x, y)}{f_Y(y)}$$
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3.6.2 – Functions of Joint Random
VariablesVariables(X, Y) have joint pdf
$$f_{XY}(x, y)$$
.We want the distribution of
 $U=g_1(X, Y), V=g_2(X, Y)$.E.g. : $U=X+Y, V=Y^2$ or $U=X+Y, V=X-2Y$, etc.STAT 702/702BiblingUniv. of S.C.

For the continuous case the joint pdf
of (U,V) is
$$f_{U,V}(u, v) = f_{X,Y}(h_1(u, v), h_2(u, v)) |J|$$

where h_1 and h_2 are the inverse
functions: $x = h_1(u, v), y = h_2(u, v))$
And J is the Jacobian $J = \begin{vmatrix} \frac{dh_1}{du} & \frac{dh_1}{dv} \\ \frac{dh_2}{du} & \frac{dh_2}{dv} \end{vmatrix}$

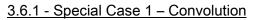


Example 1) X and Y have joint p.d.f. $f_{XY}(x,y) = 2$ $0 \le x,y \le 1$ U=X/Y and V=Y Find the joint and marginal p.d.f's of X and Y.

Example 2) X and Y are bivariate normal with means 0, variances 1, and correlation = 0.

Let
$$r = \sqrt{x^2 + y^2}$$
 and $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

Find the joint and marginal distributions.

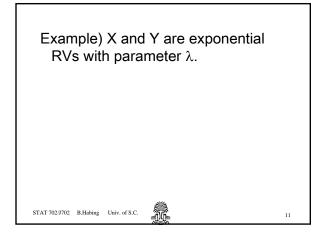


In general say Z=X+Y

We can find a general formula for $F_Z(z)=P(Z \le z)$ simply by finding the appropriate area under f(x,y).

Taking the derivative then gives us the pdf.

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3.6.1 - Special Case 2 - Quotient

A general formula for the quotient Z=Y/X can also be derived by examining the CDF.

To do this easily, note that if $y/x \le z$ then if x>0 we have $y \le xz$ and if x<0 then $y \ge xz$.

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Back to the earlier example)
X and Y have joint p.d.f.
$$f_{XY}(x,y) = 2$$
 $0 \le x < y \le 1$
U=X/Y