STAT 702/J702

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-Lecture 16-

Instructor: Brian Habing
Department of Statistics
Telephone: 803-777-3578
E-mail: habing@stat.sc.edu

Today

- Continuous Joint Distributions (continued)
- Functions of Jointly Distributed Random Variables

Example 3)

$$f(x,y)=\frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}}\exp\left\{-\frac{1}{2(1-\rho^2)}\left[\frac{(x-\mu_x)^2}{\sigma_x^2}-\frac{2\rho(x-\mu_x)(y-\mu_y)}{\sigma_x\sigma_y}+\frac{(y-\mu_y)^2}{\sigma_y^2}\right]\right\},$$

$-\infty<\mu_x, \mu_y<\infty$, $\sigma_x, \sigma_y > 0$, $-1< \rho <1$

$$f_X(x) = \int_{-\infty}^{\infty} f(x, y)dy$$

$$f_Y(y) = \int_{-\infty}^{\infty} f(x, y)dx$$

$$f_{Y|X}(y \mid x) = \frac{f_{X,Y}(x, y)}{f_X(x)}$$

$$f_{X|Y}(x \mid y) = \frac{f_{X,Y}(x, y)}{f_Y(y)}$$

3.6.2 – Functions of Joint Random Variables

(X, Y) have joint pdf $f_{XY}(x, y)$.

We want the distribution of $U=g_1(X, Y)$, $V=g_2(X, Y)$.

E.g. : U=X+Y, V=Y^2 or U=X+Y, V=X-2Y, etc.

For the continuous case the joint pdf of (U,V) is

$f_{U,V}(u, v) = f_{X,Y}(h_1(u, v), h_2(u, v))\ |J|$

where h_1 and h_2 are the inverse functions: $x = h_1(u, v)$, $y = h_2(u, v)$)

And J is the Jacobian $J = \begin{vmatrix} \frac{dh_1}{du} & \frac{dh_1}{dv} \\ \frac{dh_2}{du} & \frac{dh_2}{dv} \end{vmatrix}$

Example 1) X and Y have joint p.d.f.
$f_{XY}(x,y) = 2 \quad 0 \leq x,y \leq 1$

U=X/Y and V=Y

Find the joint and marginal p.d.f's of X and Y.

Example 2) X and Y are bivariate normal with means 0, variances 1, and correlation = 0.

Let $r = \sqrt{x^2 + y^2}$ and $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

Find the joint and marginal distributions.

3.6.1 - Special Case 1 – Convolution

In general say Z=X+Y

We can find a general formula for $F_Z(z)$=P(Z<z) simply by finding the appropriate area under $f(x,y)$.

Taking the derivative then gives us the pdf.

Example) X and Y are exponential RVs with parameter λ.

3.6.1 - Special Case 2 – Quotient

A general formula for the quotient Z=Y/X can also be derived by examining the CDF.

To do this easily, note that if y/x≤z then if x>0 we have y ≤ xz and if x<0 then y ≥ xz.

Back to the earlier example)

X and Y have joint p.d.f.

$f_{XY}(x,y) = 2 \quad 0 \le x < y \le 1$

U=X/Y