Today

• Homework

• More on Expected Values

• Moment Generating Functions

Ch 3: #66) First find the c.d.f. of $X(k)$ and then find the p.d.f. by differentiating.
Problem 2) Assume a sample of size 30 is supposed to have come from a standard normal distribution. What is the 99th-\%ile for the 30th order statistic?

Chapter 4 Revisited:
More on Expected Values

For constants a and b,

\[ E(a + b X) = a + b E(X) \]
\[ \text{Var}(a + b X) = b^2 \text{Var}(X) \]

Let \( X_1, X_2, \ldots, X_n \) be mutually independent random variables, then:

\[ \mu_{\Sigma X} = E(\Sigma_i X_i) = \Sigma_i E(X_i) = \Sigma_i \mu_{X_i} \]
\[ \sigma_{\Sigma X}^2 = \text{Var}(\Sigma_i X_i) = \Sigma_i \text{Var}(X_i) = \Sigma_i \sigma_{X_i}^2 \]
What if the $X_i$ are not independent?

First, if the $X_i$ have joint p.d.f $f(x_1,\ldots,x_n)$ and $Y = g(x_1,\ldots,x_n)$ then

\[ E(Y) = \int \cdots \int g(x_1,\ldots,x_n) f(x_1,\ldots,x_n) \, dx_1 \cdots dx_n \]

Provided the integral converges with $|g|$. 

Now consider $Y = a + b \sum_{i=1}^n X_i$

and finding $E(Y)$ and $\text{Var}(Y)$.

Covariance

\[ \text{Cov}(X,Y) = E[(X - \mu_X)(Y - \mu_Y)] \]

Correlation

\[ \text{Cor}(X,Y) = \rho_{XY} = \frac{\text{Cov}(X,Y)}{\sqrt{\text{Var}(X)\text{Var}(Y)}} \]
4.5 – Moment Generating Functions

The moment-generating function (mgf) of \( X \) is \( M(t) = E(e^{tX}) \)

\[
M(t) = \sum_x e^{tx} p(x)
\]

\[
M(t) = \int_{-\infty}^{\infty} e^{tx} f(x) dx
\]

Why “moment generating”?

Assume the mgf exists on some interval around 0…

\[
M(t) = \int_{-\infty}^{\infty} e^{tx} f(x) dx
\]

\[
M'(t) = \frac{d}{dt} \int_{-\infty}^{\infty} e^{tx} f(x) dx
\]

Other properties:

a) The m.g.f. uniquely determines the p.d.f.

b) If \( Y = a + bX \) then \( M_Y(t) = e^{at} M_X(bt) \)

c) If \( X \) and \( Y \) are independent and \( Z = X + Y \) then \( M_Z(t) = M_X(t) M_Y(t) \)