Homework 12 – Problem #2

Let $X$ and $Y$ be independent uniform [0,1] random variables.

Consider the (seemingly ugly) transformations:

$U = \sqrt{-2 \ln(X)} \cos(2\pi Y)$
$V = \sqrt{-2 \ln(X)} \sin(2\pi Y)$

a) Demonstrate that:

$X = \exp\left(-\frac{U^2 + V^2}{2}\right)$

$Y = \frac{1}{2\pi} \arctan \frac{V}{U}$

b) Use the transformation of variable formula to find the joint distribution of $U$ and $V$, and remember to specify where it is defined.

c) Identify the joint distribution by name.

Hints - Some relevant reminders from Trigonometry and Calculus

$\sin^2 \theta + \cos^2 \theta = 1$

$\frac{\sin \theta}{\cos \theta} = \tan \theta$

$\frac{d}{du} \arctan u = \frac{1}{1 + u^2} du$

and remember the chain rule!!