Let $X_1, \ldots X_n$ be a random sample from distribution $F_X(x)$, and let $V=X_{(1)}$ be the minimum value.

Now $V \leq v$ means that at least one of the $X_i$ must be less than or equal to $v$, because the minimum is one of the values (which isn’t what I said in class!).

Using the complement rule, this is the complement of all of them being greater than $v$, so $P(V \leq v) = 1 - P(V \geq v)$

Now notice that $V \geq v$ if and only if all of the values are at least that big, so…

\[
P(V \geq v) = P[(X_1 \geq v) \cap (X_2 \geq v) \cap \cdots \cap (X_n \geq v)]
\]

\[
= P(X_1 \geq v)P(X_2 \geq v) \cdots P(X_n \geq v) \quad \text{by independence}
\]

\[
= [1 - P(X_1 \leq v)] [1 - P(X_2 \leq v)] \cdots [1 - P(X_n \leq v)] \quad \text{by complement rule}
\]

\[
= [1 - F_X(v)] [1 - F_X(v)] \cdots [1 - F_X(v)] \quad \text{by defn of the CDF}
\]

\[
= [1 - F_X(v)]^n
\]

And so we have that $F_X(v) = P(V \leq v) = 1 - P(V \geq v) = 1 - [1 - F_X(v)]^n$