STAT 703/J703 – Spring 2007 - Take Home Exam 2

Due by 4:00pm, Thursday, March 29th

Answer 10 of the 11 following questions (I will grade your best 10). Show all of your work for credit.
There are no "trick" questions.
You are not to work with anyone else on this exam, or to receive assistance with it from anyone but me.

Questions 1-9 concern an independent and identically distributed random sample $X_1, \ldots X_{10}$ from a Negative Binomial distribution with parameters r =5 and p unknown.

1) Find the distribution of the sum of n independent Negative Binomials that each have parameters r and p.

2) Show that the likelihood ratio test of H_0: p=0.5 vs. H_A: p=0.8 that has α as close to 0.05 as possible without exceeding it is of the form "Reject if $\sum x_i \leq c$" where the constant c=83 corresponds to α=0.039. (Note: R's negative binomial only counts the number of failures before the r^{th} success is reached. To make it match the version used in the text you need to subtract r from your x value before putting it into `pnbinom` in R, or add r to the x value R gives you when using `qnbinom`.)

3) Apply the test from (2) to the data set: `7 5 6 9 8 9 7 9 7 7` and state whether you reject or fail to reject the null hypothesis.

4) Find the p-value for the test in (2) and briefly say how it should be interpreted.

5) Find the power of the test in (2) if the true p is 0.8 and briefly say how it should be interpreted.

6) Verify that the test found in (2) is UMP for H_0: p=0.5 vs. H_A: p>0.5.

7) Briefly explain why there is no UMP test for H_0: p=0.5 vs. H_A: p≠0.5.

8) Calculate -2logΛ for the generalized likelihood ratio test of H_0: p=0.5 vs. H_A: p≠0.5. (You may use the fact that the MLE of p for the Negative Binomial is $nr/\sum x_i$).

9) State the asymptotic distribution for the test statistic in (7), being sure to state the degrees of freedom for the distribution, and the rejection region for α=0.05. Use this to conduct the test using the data in (3).

The formula for the 100(1-α)% confidence interval for σ^2 for a random sample from a normal population is given as:

$$\left(\frac{n\hat{\sigma}^2}{\chi^2_{df=n-1,1-\alpha/2}}, \frac{n\hat{\sigma}^2}{\chi^2_{df=n-1,\alpha/2}}\right)$$

where $\hat{\sigma}^2$ is the MLE for the variance and $\chi^2_{df=n-1,\alpha}$ is the lower α^{th} percentile point for a chi-squared distribution with n-1 degrees of freedom.

10) Use the above formula to say how you would test H_0: $\sigma^2 = \sigma_0^2$ vs. H_A: $\sigma^2 \neq \sigma_0^2$.

11) Test H_0: σ^2= 2 vs. H_A: $\sigma^2 \neq$ 2 for the following sample that is from a normal distribution at the α=0.05 level.

2.26	1.40	-0.64	0.31	2.88
0.65	0.00	4.37	-0.81	1.56