STAT 703/J703
February 1st, 2007
-Lecture 6-

Instructor: Brian Habing
Department of Statistics
LeConte 203
Telephone: 803-777-3578
E-mail: habing@stat.sc.edu

Today

- Homework Solutions
- Logistic Regression Continued
- Sections 8.5.2-8.5.4: Large Sample Properties of the MLE

1) Find the formula for the method of moment estimator for a geometric random variable.

Note, for a beta distribution:

$$\hat{\alpha} = \left(\frac{\bar{x}}{\hat{\sigma}^2}\right)\left(\bar{x} - \bar{x}^2 - \hat{\sigma}^2\right) \qquad \hat{\beta} = \left(\frac{1-\bar{x}}{\hat{\sigma}^2}\right)\left(\bar{x} - \bar{x}^2 - \hat{\sigma}^2\right)$$

2) Use method of moments to estimate the beta distribution α and β parameters for column c of the data set `itests.txt`

3) Construct a q-q plot to verify if column c of the itests data seems to be from a beta distribution with the parameter estimates you found in (2)

4) Use the parametric bootstrap to estimate the bias and standard error of the method of moments estimators you found in (2).

MLE example 2 – Logistic Regression

In linear regression we try to predict one continuous variable from another.

Under a few basic assumptions this results in fairly easy parameter estimation and use of t and F distributions.

Consider the case of attempting to predict a 0,1 variable from a continuous variable.

$$P[Y_i = 1 | x_i] = \frac{1}{1 + e^{-(\alpha + \beta x_i)}}$$

8.5.2 – Large Sample Properties

Theorem A: The MLE is Consistent (under appropriate regularity conditions)

STAT 702/J702 B.Habing Univ. of S.C. 10

Information function

$$I(\theta) = E\left[\frac{\partial}{\partial\theta}\log f(X \mid \theta)\right]^2$$

$$= -E\left[\frac{\partial^2}{\partial\theta^2}\log f(X \mid \theta)\right]^2$$

STAT 702/J702 B.Habing Univ. of S.C. 11

Theorem B: Under appropriate regularity conditions the MLE is asymptotically normal with mean θ and variance $\frac{1}{nI(\theta)}$.

STAT 702/J702 B.Habing Univ. of S.C. 12