STAT 703/J703
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-Lecture 8-

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Today

- Homework Solutions
- Information Function Continued
- Asymptotic Normality of the MLE

1) Find the MLE for the geometric distribution parameter p.

2a) Briefly explain why the code needs to use the lines defining s and m. Be sure to include any math needed to show why that is correct.

b) Give a reason why using the value b=0 would be a bad choice for an initial value estimate.

c) Use the resulting estimated logistic regression equation to estimate the probability of O-ring failure at a temperature of 29 degrees.

d) Imagine you were attempting to use logistic regression on something where an increase in X corresponded to a decrease in Y. What is one way you could modify the the code or the data to allow you to estimate this.

Information function

$$I(\theta) = E\left[\left(\frac{\partial}{\partial\theta}\log f(X\mid\theta)\right)^2\right]$$

$$= -E\left[\frac{\partial^2}{\partial\theta^2}\log f(X\mid\theta)\right]$$

Example 2: Consider a random sample of size n from a normal distribution with unknown mean θ and known variance σ^2.

What if there is more than one parameter?

In this case you get an information matrix:

$$I(\theta) = E\left[\left(\frac{\partial}{\partial \theta_i}\log f(X|\theta)\right)\left(\frac{\partial}{\partial \theta_j}\log f(X|\theta)\right)\right]$$

$$= -E\left[\frac{\partial^2}{\partial \theta_i \partial \theta_j}\log f(X|\theta)\right]$$

$$\frac{\partial}{\partial \mu}\log f(X|\mu,\sigma) = \frac{(x-\mu)}{\sigma^2}$$

$$\frac{\partial}{\partial \sigma}\log f(X|\mu,\sigma) = -\frac{1}{\sigma} + \frac{(x-\mu)^2}{\sigma^3}$$

$$\frac{\partial^2}{\partial \mu^2}\log f(X|\mu,\sigma) = \frac{-1}{\sigma^2}$$

$$\frac{\partial^2}{\partial \sigma^2}\log f(X|\mu,\sigma) = \frac{1}{\sigma^2} - \frac{3(x-\mu)^2}{\sigma^4}$$

$$\frac{\partial^2}{\partial \mu \partial \sigma}\log f(X|\mu,\sigma) = -\frac{2(x-\mu)}{\sigma^3}$$

Theorem B: Under appropriate regularity conditions the MLE is asymptotically normal with mean θ and variance $\frac{1}{nI(\theta)}$.

How does that work for the Poisson λ?
(Recall we found that $\hat{\lambda}_{mle} = \overline{x}$
$I(\lambda) = 1/\lambda$)

How does that work for the $N(\mu,\sigma^2)$?

What went wrong?

Sketch of proof: Consider the Taylor series expansion:

$$L'(\hat{\theta}) \approx L'(\hat{\theta}) + (\hat{\theta} - \theta)L''(\hat{\theta})$$

Confidence Interval Construction:

1) Exact Methods

2) Asymptotic Approximations

3) Bootstrap