Today

• Basics of Hypothesis Testing
  Continued

9.2 Neyman-Pearson Paradigm
Let $X = (X_1, ..., X_n)$ denote a sample from population $f(x|\theta)$. Decide on $H_0$ vs. $H_A$ based on the sample.

A decision on whether or not to reject $H_0$ in favor of $H_A$ is made on the basis of a statistic

$$T = T(X) = T(X_1, ..., X_n).$$
The set of values of $T$ for which $H_0$ is accepted is called the acceptance region and the set of values of $T$ for which $H_0$ is rejected is the rejection region of the test.

Two kinds of error may occur:

1. $H_0$ is rejected when it is true: Type I error.
   
   $P$(type I error) = $\alpha$
   
   $\alpha = P(T \in$ rejection region $| H_0$ true).

   If $H_0$ is simple, $\alpha$ is called the significance level of the test.

2. $H_0$ is accepted when it is false: Type II error.
   
   $P$(type II error) = $\beta$
   
   $\beta = P(T \in$ acceptance region $| H_0$ false)

   If $H_A$ is composite, $\beta$ depends on which member of $H_A$ is the true pdf.
Power of the test = \( P(H_0 \text{ is rejected when false}) \)
= 1 - \( P(H_0 \text{ is accepted | } H_0 \text{ false}) \)
= 1 - \( \beta \).

Ideally, we would want \( \alpha = \beta = 0 \), but this not possible since the decision is based on data.

Example:
Consider testing
\( H_0: p = 0.5 \)
vs. \( H_A: p = 0.6 \)
for a binomial sample of size \( n = 10 \).

P-value The p-value is the probability of observing a test statistic at least as extreme as the one observed if the null hypothesis is true.

The null hypothesis is rejected when p-value is \( \leq \alpha \). It is the smallest \( \alpha \) for which \( H_0 \) would be rejected.
Example 2:

Consider testing
$H_0: \ p=0.5$
vs. $H_A: \ p>0.5$
for a binomial sample of size $n=10$.

For a composite test the significance level $\alpha$ is the maximum (supremum) of the probabilities of a Type I error over all the possible alternatives.