Today

• 9.3: Neyman-Pearson Lemma

• 9.4: Tests and CIs

Neyman-Pearson Lemma: If the likelihood ratio test that rejects $H_0$ in favor of $H_A$ when

$$\lambda = \frac{f_A(x)}{f_0(x)} \leq c,$$

has significance level $\alpha$, then any other test having significance level at most $\alpha$ has power less than or equal to the power of the likelihood ratio test. (i.e., the LRT has highest power among tests with significance level $\alpha$).
Example: Consider a sample of size \( n \) from a normal distribution with variance 1.

Test \( H_0: \mu = 0 \) vs. \( H_A: \mu = 1 \) at \( \alpha = 0.05 \).

In some cases we can also show that the test is uniformly most powerful for a composite alternate hypotheses.

This happens if we can show it is most powerful for every simple alternate in \( H_A \).

Consider testing

Test \( H_0: \mu = 0 \) vs. \( H_A: \mu > 0 \)

and

Test \( H_0: \mu = 0 \) vs. \( H_A: \mu \neq 0 \)
Example 2: Consider a binomial distribution with \( n=8 \) and unknown \( p \). It is desired to test \( H_0: p=0.2 \) versus \( H_A: p=0.4 \).

Confidence Intervals and Tests

There is a duality between confidence intervals and hypothesis tests. A confidence interval is found by “inverting” a two-sided test (and vice-versa).

Theorem A: Suppose there is a test of level \( \alpha \) for \( H_0: \theta = \theta_0 \), and let \( A(\theta_0) \)=acceptance region

Then the set \( C=\{\theta: X \in A(\theta)\} \) is a \( 100(1-\alpha)\% \) confidence region for \( \theta \).
Theorem B: Let \( C(X) \) be a \( 100(1- \alpha) \% \) confidence region for \( \theta_0 \).

Then \( A(\theta_0) = \{ X: \theta_0 \in C(X) \} \) is an acceptance region for a test of level \( \alpha \) for \( H_0: \theta = \theta_0 \).