STAT 703/J703
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-Lecture 13-

Instructor: Brian Habing
Department of Statistics
LeConte 203
Telephone: 803-777-3578
E-mail: habing@stat.sc.edu

Today

- Exam 1 Solutions
- Neyman-Pearson Lemma Examples Continued

Questions 1-7 deal with the distribution that has pdf $f(x)=(1/\theta)x^{(1-\theta)/\theta}$ for $0 < x < 1$ where $\theta > 0$. Note that this is actually a Beta distribution with parameters $a=1/\theta$ and $b=1$.

Below is a random sample of 14 that supposedly comes from such a distribution.

0.01	0.11	0.19	0.34	0.34
0.37	0.41	0.43	0.76	0.77
0.81	0.82	0.83	0.89	

1) Find the formula for the method of moments estimator for θ and its value for the above sample.

2) Find the formula for the maximum likelihood estimator for θ and its value for the above sample.

3) Find the asymptotic variance of the mle of θ.

4) Construct an approximate 95% confidence interval for θ for the above data set using the asymptotic properties of the mle.

5) Use the parametric bootstrap to estimate the bias and standard deviation of the mle for θ for a sample of size 10 from this distribution with the value of θ you found in (2).

6) Construct a q-q plot to check if the sample seems to come from this type of distribution. Does it? If not, briefly describe why not in terms of how the data appears in comparison to the shape of the distribution.

7) Imagine that someone believes the above data set comes from a beta distribution, but not one where β=1. In this case we might want to find the MLE's for both α and β for this data set. This could be annoying to do because the pdf involves gamma functions. We could still find the answer numerically using R. Use **`optim`** to estimate the mle.

Questions 8-9 consider the shifted exponential distribution that has pdf $f(x)=\lambda e^{-\lambda(x-\alpha)}$ where $\alpha \leq x < \infty$. This distribution has mean $\alpha + (1/\lambda)$ and variance $1/\lambda^2$.

8) Find the method of moments estimators for this distribution.

9) Find the maximum likelihood estimators for this distribution.

10) Having the properties of consistency and asymptotic normality are often viewed as very important properties of estimators by statisticians. In one sentence, explain why these properties are not particularly useful for analyzing the data set in 1-7.

11) Many introductory statistics texts make an analogy between hypothesis testing and a criminal trial. They also say that it is bad to "accept" the null hypothesis because if you tested it again with more data you might then be able to reject it. What concept would this gathering of more data after the test be analogous to in the criminal trial? Is it allowed?

<u>Example Continued:</u>

Consider a sample $X_1, \ldots X_n$ from a binomial distribution with known parameter m and unknown parameter p.

What is the most powerful test of the hypothesis H_0: p = .5 vs. H_A: p = .6?