STAT 703/J703
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-Lecture 14-

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Today

- 9.4: Generalized Likelihood Ratio Test
- 9.3: Duality Between Tests and Confidence Intervals

<u>9.4 Generalized Likelihood Ratio Tests</u>

The Neyman-Pearson likelihood ratio test is most powerful for simple vs. simple.

Here, we generalize to composite hypotheses. The generalized LRT is not necessarily optimal, but works well for situations where no optimal test exists.

Observe $X_1, \ldots, X_n$ from $f(x|\theta)$.
Test H_0: $\theta \in \omega_0$ $\quad \omega_0 \subset \Omega$
vs. H_A: $\theta \in \omega_1$ $\quad (\omega_0 \cup \omega_1 = \Omega)$.
Use the generalized LR statistic.

$$\Lambda = \frac{\max\limits_{\theta \in \omega_0} \left(lik(\theta)\right)}{\max\limits_{\theta \in \Omega} \left(lik(\theta)\right)} = \frac{\max\limits_{\theta \in \omega_0} \prod_{i=1}^{n} f(x_i \mid \theta)}{\max\limits_{\theta \in \Omega} \prod_{i=1}^{n} f(x_i \mid \theta)}$$

Reject H_0 if $\Lambda \le \lambda_0$, where λ_0 is
P(rej. $H_0 | \theta \in \omega_0$)$=\alpha$.

(Note: If H_0 holds, $\Lambda=1$. If H_A holds, $\Lambda<1$, small).

Use this to construct the test, i.e. find rejection regions in terms of simple statistics (similar to N-P lemma).

Example: Consider a random sample from a normal distribution with unknown mean and unknown variance.

Theorem A pg.341: Under smoothness conditions on the pdf, the null distribution of $-2\ln\Lambda$ has an approximate chi-square distribution with d.f.=$\dim\Omega - \dim\omega_0$ for large n.

$N(\mu,\sigma^2)$, both unknown
$\Rightarrow$ df = 2-1 = 1.

9.3 The Duality of Confidence Intervals and Hypothesis Tests

There is a duality between confidence intervals and hypothesis tests. A confidence interval is found by "inverting" a two-sided test (and vice-versa).

Theorem A, pg. 338: Suppose there is a test of level α for H_0: $\theta = \theta_0$, and let $A(\theta_0)$=acceptance region

Then the set $C=\{\theta: \underline{X} \in A(\theta)\}$ is a $100(1-\alpha)\%$ confidence region for θ.

Theorem B, pg. 338: Let $C(\underline{X})$ be a $100(1-\alpha)\%$ confidence region for θ_0.

Then $A(\theta_0)=\{\underline{X}: \theta_0 \in C(\underline{X})\}$ is an acceptance region for a test of level α for H_0: $\theta = \theta_0$

Example cont.: Again, consider the random sample from a normal distribution with unknown mean and unknown variance.