Today

- Homework 4
- 9.5: Generalized Likelihood Ratio Test Revisited
- 9.4: Duality Between Tests and Confidence Intervals

4) A new type of product is supposed to have a mean time until failure of at least 5 hours. A sample of size 8 produced first failure times of 1.6, 4.3, 4.7, 5.8, 6.3, 2.1, 8.5, and 3.2. Determine the appropriate null and alternate hypotheses to determine if the producer should be alerted that they aren't lasting long enough, then set up and perform the best test.
9.5 Generalized Likelihood Ratio Tests

Observe $X_1,\ldots, X_n$ from $f(x|\theta)$. Test $H_0: \theta \in \omega_0$ vs. $H_A: \theta \in \omega_1$.

Use the generalized LR statistic.

\[
\Lambda = \max_{\theta \in \omega_0} \left(\frac{\text{lik} (\theta)}{\text{lik}}\right) \max_{\theta \in \Omega} \frac{\prod_{i=1}^{n} f(x_i | \theta)}{}\max_{\theta \in \omega_1} \frac{\prod_{i=1}^{n} f(x_i | \theta)}{\text{lik} (\theta)}
\]

Reject $H_0$ if $\Lambda \leq \lambda_0$, where $\lambda_0$ is

\[P(\text{rej. } H_0|\theta \in \omega_0) = \alpha.\]

(Note: If $H_0$ holds, $\Lambda = 1$. If $H_A$ holds, $\Lambda < 1$, small).

Use this to construct the test, i.e. find rejection regions in terms of simple statistics (similar to N-P lemma).

Example: Consider a random sample from a normal distribution with unknown mean and unknown variance.
Theorem A pg.310: Under smoothness conditions on the pdf, the null distribution of $-2\ln \Lambda$ has an approximate chi-square distribution with d.f.=$\dim \Omega - \dim \omega_0$ for large $n$.

$\mathcal{N}(\mu, \sigma^2)$, both unknown and $H_0: \mu = \mu_0$ \Rightarrow df = 2-1 = 1.

9.4 The Duality of Confidence Intervals and Hypothesis Tests

There is a duality between confidence intervals and hypothesis tests. A confidence interval is found by “inverting” a two-sided test (and vice-versa).

Theorem A, pg. 307: Suppose there is a test of level $\alpha$ for $H_0: \theta = \theta_0$, and let $A(\theta_0)=$ acceptance region.

Then the set $C=\{\theta: X \in A(\theta)\}$ is a $100(1-\alpha)%$ confidence region for $\theta$. 
Theorem B, pg. 307: Let \( C(X) \) be a 100(1 - \( \alpha \))% confidence region for \( \theta_0 \).

Then \( A(\theta_0) = \{X: \theta_0 \in C(X)\} \) is an acceptance region for a test of level \( \alpha \) for \( H_0: \theta = \theta_0 \).

Example cont.: Again, consider the random sample from a normal distribution with unknown mean and unknown variance.