Today

• Power of a Test (Cont.)

• Example: Test for a Multinomial Distribution

Power: $P(\text{reject } H_0 \mid H_0 \text{ is False})$
Non-central t-distribution

\[ t_{df, \Delta} = \frac{Z + \Delta}{\sqrt{\frac{X^2}{df}} \sqrt{\frac{s}{\sqrt{n}}} = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}} \]

then

\[ \Delta = \frac{\mu_A - \mu_0}{\sigma / \sqrt{n}} \]

Example: Consider a data set that could have come from a binomial distribution with n=5, but may also have come from a hypergeometric or some other distribution.

<table>
<thead>
<tr>
<th>X</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>#obs</td>
<td>9</td>
<td>21</td>
<td>16</td>
<td>10</td>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>

Test H₀: X is binomial vs. Hₐ: it isn’t

So in general we get

\[ G^2 = -2 \log \Lambda = 2 \sum_{i=1}^{n} O_i \log \left( \frac{O_i}{E_i} \right) \]
Page 342-343 demonstrates that this is asymptotically the same as the classic

\[ \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i} \]

Theorem A pg.310: Under smoothness conditions on the pdf, the null distribution of \(-2\ln \Lambda\) has an approximate chi-square distribution with d.f. = \(\text{dim} \Omega - \text{dim} \omega_0\) for large n.

Note: You must be careful what estimates you use for the parameters!!!