Today

• The End!

• 9.5: Contingency Table
  Example

• 9.4: Duality Between Tests and Confidence Intervals

9.5 Generalized LRT

Observe $X_1, \ldots, X_n$ from $f(x|\theta)$.

Test $H_0$: $\theta \in \omega_0$ $\omega_0 \subset \Omega$

vs. $H_A$: $\theta \in \omega_1$ ($\omega_0 \cup \omega_1 = \Omega$).

$$
\Lambda = \frac{\max_{\theta \in \omega_0} (\text{lik}(\theta))}{\max_{\theta \in \Omega} (\text{lik}(\theta))} = \frac{\max_{\theta \in \omega_0} \prod_{i=1}^{n} f(x_i | \theta)}{\max_{\theta \in \Omega} \prod_{j=1}^{n} f(x_j | \theta)}
$$

Reject $H_0$ if $\Lambda \leq \lambda_0$, where $\lambda_0$ is
Theorem A pg.310: Under smoothness conditions on the pdf, the null distribution of \(-2\ln\Lambda\) has an approximate chi-square distribution with d.f. = \(\dim\Omega - \dim\omega_0\) for large \(n\).

Example: Consider a data set that could have come from a binomial distribution with \(n=5\), but may also have come from a hypergeometric or some other distribution.

<table>
<thead>
<tr>
<th>(X)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>#obs</td>
<td>9</td>
<td>21</td>
<td>16</td>
<td>10</td>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>

Test \(H_0: X\) is binomial vs. \(H_A: X\) isn’t

So in general we get

\[
G^2 = -2\log\Lambda = 2\sum_{i=1}^{n} O_i \log\left(\frac{O_i}{E_i}\right)
\]

Page 311-312 demonstrates that this is asymptotically the same as the classic

\[
\sum_{i=1}^{n} \left(\frac{O_i - E_i}{E_i}\right)^2
\]
Note: You must be careful what estimates you use for the parameters!!!

9.4 The Duality of Confidence Intervals and Hypothesis Tests

There is a duality between confidence intervals and hypothesis tests. A confidence interval is found by “inverting” a two-sided test (and vice-versa).

Theorem A, pg. 307: Suppose there is a test of level $\alpha$ for $H_0: \theta = \theta_0$, and let $A(\theta_0) =$ acceptance region

Then the set $C=\{\theta: X \in A(\theta)\}$ is a $100(1- \alpha)$% confidence region for $\theta$. 
Theorem B, pg. 307: Let $C(X)$ be a $100(1 - \alpha)\%$ confidence region for $\theta_0$.

Then $A(\theta_0) = \{X: \theta_0 \in C(X)\}$ is an acceptance region for a test of level $\alpha$ for $H_0: \theta = \theta_0$.

Example cont.: Consider the random sample from a normal distribution with unknown mean and unknown variance.