Today

- Homework 5 Solutions
- 9.4: Duality Between Tests and Confidence Intervals

Chapter 9 #6: Develop a likelihood ratio test of $H_0: \theta=0.6$ versus $H_A: \theta=0.7$ based on $n=10$ trials.

(Follow the reasoning of Example A of Section 9.3)
Chapter 9 #18: True or False

a) The generalized likelihood ratio statistic $L$ is always less than or equal to 1.

b) If the p-value is 0.03, the corresponding test will reject at the significance level 0.02.

c) If a test rejects at significance level 0.06, then the p-value is less than or equal to 0.06.

d) The p-value of a test is the probability that the null hypothesis is correct.

e) In testing a simple versus simple hypothesis via the likelihood ratio, the p-value equals the likelihood ratio.

f) If a chi-square test-statistic with 4 degrees of freedom has a value of 8.5, the p-value is less than 0.05.
9.4 The Duality of Confidence Intervals and Hypothesis Tests

There is a duality between confidence intervals and hypothesis tests. A confidence interval is found by “inverting” a two-sided test (and vice-versa).

Theorem A, pg. 307: Suppose there is a test of level $\alpha$ for $H_0: \theta = \theta_0$, and let $A(\theta_0)$=acceptance region

Then the set $C=\{\theta: X \in A(\theta)\}$ is a $100(1-\alpha)$% confidence region for $\theta$.

Theorem B, pg. 307: Let $C(X)$ be a $100(1-\alpha)$% confidence region for $\theta_0$.

Then $A(\theta_0)=\{X: \theta_0 \in C(X)\}$ is an acceptance region for a test of level $\alpha$ for $H_0: \theta = \theta_0$. 
Example: Consider the random sample from a normal distribution with unknown mean and unknown variance.

Chapter #11.2 Let $X_1, \ldots, X_n$ be an independent random sample from a population that is normal with mean $\mu_x$ and variance $\sigma^2$, and let $Y_1, \ldots, Y_m$ be an independent random sample from a population that is normal with mean $\mu_y$ and variance $\sigma^2$, such that both samples are independent. Consider testing about the means.

The likelihood ratio statistic says to reject for large values of

$$1 + \frac{mn}{m+n} \left( \frac{(\bar{x} - \bar{y})^2}{\sum_{i=1}^{n} (x_i - \bar{x})^2 - \sum_{i=1}^{n} (y_i - \bar{y})^2} \right)$$