STAT 703/J703
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-*Lecture 18*-

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Today

- Homework Solutions
- Example 1: Test for a Multinomial Distribution (cont.)
- Example 2: Two Sample Tests for Location

Homework 5

A new type of product is supposed to have a mean time until failure of at least 5 hours. A sample of size 8 produced first failure times of 1.6, 4.3, 4.7, 5.8, 6.3, 2.1, 8.5, and 3.2.

a) Assuming the failure times follow an exponential distribution, find the UMP test for H0:mean failure time=5 hours versus HA: mean failure time < 5 hours. (Remember that the expected value is 1/lambda).

b) Find the p-value for testing these hypotheses with the given data set. (Hint: pgamma)

c) Find the power for testing these hypotheses when the true mean failure time is 4 hours.

Example 1: Consider a data set that could have come from a binomial distribution with n=5, but may also have come from a hypergeometric or some other distribution.

X	0	1	2	3	4	5
#obs	9	21	16	10	4	0

Test H_0: X is binomial vs. H_A: it isn't

So in general we get

$$G^2 = -2\log\Lambda = 2\sum_{i=1}^{n} O_i \log\left(\frac{O_i}{E_i}\right)$$

Which has an approximate chi-square distribution with d.f.=dimΩ-dimω_0 for large n.

Example 1b: Consider the case where the data is supposedly from a fair die with sides numbered 0, 1, 2, 3, 4, 5.

Example 1c: Consider the case where the data is supposedly from an exponential random variable and rounded to the nearest integer.

Example 2 Let $X_1, \ldots X_n$ be an independent random sample from a population that is normal with mean μ_x and variance σ^2, and let $Y_1, \ldots Y_m$ be an independent random sample from a population that is normal with mean μ_Y and variance σ^2, such that both samples are independent.

Consider testing about the means.

The likelihood ratio statistic statistic says to reject for large values of

$$1+\frac{mn}{m+n}\left(\frac{(\bar{x}-\bar{y})^2}{\sum_{i=1}^{n}(x_i-\bar{x})^2-\sum_{i=1}^{n}(y_i-\bar{y})^2}\right)$$

What if the data is the data is not normal?

What if we don't know the distribution of the data?

In this case a new paradigm must be used.

Many procedures in statistics are based on replacing the data with ranks.

In the Mann-Whitney-Wilcoxon test all of the data in both groups are replaced by their ranks from smallest to largest.

The population that tends to have larger values will have more large ranks.

Another way to look at it is by considering if the probability that a randomly chosen member of one group is less than a randomly chosen member of another group.

You can show that the number of pairs where $X<Y$ is equal to the sum of the ranks of $Y - m(m+1)/2$.