So in general we get

\[ G^2 = -2 \log \Lambda = 2 \sum_{i=1}^{n} O_i \log \left( \frac{O_i}{E_i} \right) \]

Which has an approximate chi-square distribution with d.f. = \( \text{dim} \Omega - \text{dim} \omega_0 \) for large \( n \).

**Example 1b:** Consider the case where the data is supposedly from a fair die with sides numbered 0, 1, 2, 3, 4, 5.

**Example 1c:** Consider the case where the data is supposedly from an exponential random variable and rounded to the nearest integer.
Example 2 Let $X_1, \ldots, X_n$ be an independent random sample from a population that is normal with mean $\mu_x$ and variance $\sigma^2$, and let $Y_1, \ldots, Y_m$ be an independent random sample from a population that is normal with mean $\mu_y$ and variance $\sigma^2$, such that both samples are independent.

Consider testing about the means.

The likelihood ratio statistic statistic says to reject for large values of

$$\frac{1 + \frac{mn}{m+n} \left( \frac{(\bar{x} - \bar{y})^2}{\sum_{i=1}^n (x_i - \bar{x})^2 - \sum_{j=1}^m (y_j - \bar{y})^2} \right)}$$

What if the data is not normal?

What if we don’t know the distribution of the data?

In this case a new paradigm must be used.
Many procedures in statistics are based on replacing the data with ranks. In the Mann-Whitney-Wilcoxon test all of the data in both groups are replaced by their ranks from smallest to largest. The population that tends to have larger values will have more large ranks.

Another way to look at it is by considering if the probability that a randomly chosen member of one group is less than a randomly chosen member of another group. You can show that the number of pairs where $X < Y$ is equal to the sum of the ranks of $Y - \frac{m(m+1)}{2}$. 