# STAT 703/J703
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*-Lecture 20-*

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## Exam 2 #1

Questions 1-9 concern an independent and identically distributed random sample $X_1,\ldots X_{10}$ from a Negative Binomial distribution with parameters $r$=5 and $p$ unknown.

1) Find the distribution of the sum of $n$ independent Negative Binomials that each have parameters $r$ and $p$.



2) Show that the likelihood ratio test of $H_0$: $p$=0.5 vs. $H_A$: $p$=0.8 that has $\alpha$ as close to 0.05 as possible without exceeding it is of the form "Reject if $\sum x_i \le c$" where the constant $c$=83 corresponds to $\alpha$=0.039. (Note: R's negative binomial only counts the number of failures before the $r$th success is reached. To make it match the version used in the text you need to subtract $r$ from your $x$ value before putting it into `pnbinom` in R, or add $r$ to the $x$ value R gives you when using `qnbinom`.)

3) Apply the test from (2) to the data set:

7 5 6 9 8 9 7 9 7 7

and state whether you reject or fail to reject the null hypothesis.

4) Find the p-value for the test in (2) and briefly say how it should be interpreted.

5) Find the power of the test in (2) if the true $p$ is 0.8 and briefly say how it should be interpreted.



6) Verify that the test found in (2) is UMP for $H_0$: $p$ =0.5 vs. $H_A$: $p$ >0.5.

7) Briefly explain why there is no UMP test for $H_0$: $p$ =0.5 vs. $H_A$: $p$ ≠0.5.



8) Calculate -2log$\Lambda$ for the generalized likelihood ratio test of $H_0$: $p$=0.5 vs. $H_A$: $p$ ≠0.5. (You may use the fact that the MLE of $p$ for the Negative Binomial is $nr/\sum x_i$ ).

9) State the asymptotic distribution for the test statistic in (7), being sure to state the degrees of freedom for the distribution, and the rejection region for $\alpha$=0.05. Use this to conduct the test using the data in (3).

The formula for the 100(1-$\alpha$)% confidence interval for $\sigma^2$ for a random sample from a normal population is given as:

$$\left( \frac{n\hat{\sigma}^2}{\chi^2_{df=n-1,1-\alpha/2}}, \frac{n\hat{\sigma}^2}{\chi^2_{df=n-1,\alpha/2}} \right)$$

10) Use the above formula to say how you would test $H_0$: $\sigma^2 = \sigma_0^2$ vs. $H_A$: $\sigma^2 \neq \sigma_0^2$.

11) Test $H_0$: $\sigma^2 = 2$ vs. $H_A$: $\sigma^2 \neq 2$ for the following sample that is from a normal distribution at the $\alpha$=0.05 level.

| | | | | |
|---|---|---|---|---|
| 2.26 | 1.40 | -0.64 | 0.31 | 2.88 |
| 0.65 | 0.00 | 4.37 | -0.81 | 1.56 |