STAT 703/J703
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-Lecture 23 & 24-

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Today

Properties of Estimators cont.

- Factorization Theorem
- Rao-Blackwell
- Lehmann-Scheffe
- Completeness
- Exponential Families

Comparing Estimates and Tests

One of the standard tools for evaluating an estimate is the mean squared error:

$$MSE(\hat{\theta}) = E(\hat{\theta} - \theta_0)^2$$
$$= Var(\hat{\theta}) + [E(\hat{\theta}) - \theta_0]^2$$

Cramer-Rao Inequality

Let X_1, … X_n be I.I.d. with density $f(x|\theta)$, and T be an unbiased estimate of θ. Then under appropriate smoothness assumptions on f

$$Var(T) \geq \frac{1}{nI(\theta)}$$

<u>Defn</u> A statistic $T(X_1, \ldots X_n)$ is sufficient for θ if the conditional distribution of X_1, … X_n given T=t does not depend on θ for any value of t.

The Factorization Theorem

A necessary and sufficient condition for T to be sufficient for θ is that the joint p.d.f. factors in the form:

$$f(x_1,\ldots x_n \mid \theta) = g[T(x_1,..x_n),\theta]h(x_1,\ldots x_n)$$

Rao-Blackwell Theorem

Let $\hat{\theta}$ be an estimator of θ with $E(\hat{\theta})^2<\infty$ for all θ. If T is sufficient for q then $\tilde{\theta}=E(\hat{\theta}\,|\,T)$ satisfies

$$E(\tilde{\theta}-\theta)^2 \le E(\hat{\theta}-\theta)^2$$

for all θ.

Example : Consider trying to estimate λ for a Poisson distribution using only X_1.

Lehmann-Scheffe Theorem

Let $\hat{\theta}$ be an unbiased estimator of θ with $E(\hat{\theta})^2<\infty$ for all θ. If T is complete sufficient for θ then $\tilde{\theta}=E(\hat{\theta}\,|\,T)$ is the uniformly minimum variance unbiased estimate (UMVUE) of θ.

Complete? A statistic T is complete for θ if the zero function is the only function that satisfies:

$$E_\theta[g(T)] = 0 \text{ for all } \theta$$

However we have a result similar to the factorization theorem for "exponential families".

$$f(\underline{x} \mid \underline{\theta}) = \exp[\sum_{i=1}^{k} T_i(\underline{x}) c_i(\underline{\theta}) + d(\underline{\theta}) + S(\underline{x})]$$

where $\underline{\theta} = (\theta_1, \ldots \theta_k)$

Under appropriate regularity conditions the vector of T's is complete sufficient.