

STAT 703/J703
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-Lecture 28-

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Homework Q1:

Find the minimum variance that any unbiased estimator of λ for an exponential distribution can have.

Chapter 8 #75:

Show that the gamma distribution belongs to the exponential family. Identify the natural parameters and sufficient statistic.

Chapter 9 #36:

Is there any evidence that the suicide rates varies seasonally, or are the data consistent with the hypothesis that this rate is a constant.

Bayesian Statistics

Classical Setup

θ is an unknown constant

X_1, \dots, X_n are observed from $f(x|\theta)$

Bayesian Setup

Θ is an unknown random variable, unobservable with pdf $g(\theta)$

X_1, \dots, X_n are observed from the conditional pdf $f(x|\theta)$

A Bayesian Statistical Model:

1. An observed sample X_1, \dots, X_n , $(X_1, \dots, X_n) = \underline{X} \in \mathcal{X}$, the data space.
2. An unobserved random vector $\underline{\theta}$ of parameters, $\underline{\theta} \in \Omega$, the parameter space.
3. The conditional density of X_1, \dots, X_n , given $\underline{\theta} = \theta$, $f(\underline{x}|\theta)$.
4. The (marginal) pdf of $\underline{\theta}$, $g(\theta)$, the prior distribution of $\underline{\theta}$.

Bayes Rule can be written

$$h(\theta|x) = \frac{f(x|\theta)g(\theta)}{\int f(x|\theta)g(\theta)d\theta}$$

Say we knew $f(x|\theta)$ and $g(\theta)$ and wanted to find the maximum likelihood estimate of $h(\theta|x)$. Why can we just find the value of θ that maximizes $f(x|\theta)g(\theta)$ and not worry about the bottom integral?

Hence, inferences are described in terms of the posterior distribution of Θ , the conditional pdf of Θ , given $\underline{X}=(x_1, \dots, x_n)$, $h(\theta|x)$. (Interpretation of posterior).

We consider Bayesian point and interval estimators as well as Bayesian tests (decisions). The point estimate is given by a Bayes rule.

Example - Sample X_1, \dots, X_n from a normal population whose variance is 1 and mean μ is a value of a random variable M ,

$$X_i|M=\mu \sim N(\mu, 1),$$

$$f(x|\mu) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2}}, \mu \in R$$

Suppose $M \sim N(\alpha, \tau^2)$ as the prior pdf.

The “Bayesian confidence interval” is called a credibility interval:

$$(\theta_0, \theta_1) \text{ where } \int_{\theta_0}^{\theta_1} h(\theta | x) d\theta = 1 - \alpha$$



“Bayesian hypothesis testing” depends on a loss function evaluated for competing hypotheses.


