STAT 740 - Spring 2004 - Homework 4

Due: Friday, March 19th

1) The function `BCamedci` puts bounds on `ahat` (the acceleration $\hat{a}$) so that the calculated lower confidence limit is less than the upper confidence limit. The bounds were determined by looking at the relationship for $\alpha_1 < \alpha_2$:

$$\alpha_1 = \Phi\left(\hat{z}_0 + \frac{\hat{z}_0 + z_{\alpha/2}}{1-\hat{a}\left(\hat{z}_0 + z_{\alpha/2}\right)}\right) < \Phi\left(\hat{z}_0 + \frac{\hat{z}_0 + z_{1-\alpha/2}}{1-\hat{a}\left(\hat{z}_0 + z_{1-\alpha/2}\right)}\right) = \alpha_2$$

and realizing that the relationship will hold if the two denominators are positive. Firstly, verify that the bounds used in `BCamedci` guarantee that the denominators are positive assuming that $\hat{z}_0 + z_{\alpha/2}$ is negative and $\hat{z}_0 + z_{1-\alpha/2}$ is positive. Secondly, for α=0.05, determine the largest and smallest values of $\frac{\#\left\{T^* < T\right\}}{B}$ for which this fix will work.

2) A fairly weak bound on the sample standard deviation is: $s \le \sqrt{\frac{n}{(n-1)}}\left(\frac{range}{2}\right)$

Now consider the case of a population with minimum value 0 that is heavily skewed to the right. We could solve the above equation for how large the range (or almost equivalently, the largest value) of the sample data set needs for it to even be possible for the sample standard deviation to achieve the population standard deviation. It might happen that because of your original sample that it will be impossible for any of the bootstrap samples to have standard deviations as large as the true one. If this happens the CI for the standard deviation won't even be close to covering.

Consider samples of size 10 from a gamma distribution with mean 1 and standard deviation 10. What is the probability that the largest of the 10 observations will be less than the needed bound? (Hint: `pgamma` can be used to find the probability for a single observation, and basic probability can give you for all 10.)

3) Create an R function `secant`, similar to the `bisect` and `Newton` functions from in class, to perform the univariate secant method to find the root of an equation.