STAT 740 - Spring 2004 - Homework 8
Due: Wednesday, April 28th

1) Construct a transition matrix for a Markov chain on the state space {1, 2, 3, 4, 5} such that it has a closed class of period 2, and a non-closed class of period 3. Find a stationary distribution for the Markov chain you constructed.

2) Demonstrate that the Markov chain specified by the transition matrix

0.2	0.8	0
0	0.3	0.7
1	0	0

is ergodic. Find its limiting distribution both by solving for it directly as a stationary distribution, and by using matrix multiplication from the equally weighted initial distribution (find the n needed for it to converge to an accuracy of within 1e-10).

3) Consider the AR(1) time series model $X_n=\alpha X_{n-1}+\varepsilon_n$ where the ε_n are i.i.d. $N(0,\sigma_2)$. Explain why this model satisfies the Markov property for a continuous state space.

4) Consider the Metropolis-Hastings estimation of the Cauchy location parameter with normal prior from in class. Instead of using a Cauchy random variable to choose the candidate-generating density we could have used a normal random variable with mean set to the previous value and a variety of standard deviations. Run this procedure three times each using normals with standard deviation 1, 10, and 100 (for a total of nine runs). Comment on the performance of each and briefly speculate on why you saw the patterns you did.