STAT 515 - Fall 2003 - Solutions to the Practice Homework

Pg. 170: 3.124 a
a) \[ \frac{20!}{10!6!4!} = \frac{20\times19\times18\times17\times16\times15\times14\times13\times12\times11\times10\times9\times8\times7\times6\times5\times4\times3\times2\times1}{10\times9\times8\times7\times6\times5\times4\times3\times2\times1 \times 6\times5\times4\times3\times2\times1 \times 4\times3\times2\times1} \]
\[ = \frac{20\times19\times18\times17\times16\times15\times14\times13\times12\times11}{6\times5\times4\times3\times2\times1 \times 4\times3\times2\times1} = \frac{4\times19\times3\times17\times4\times5\times7\times13\times1\times11}{2} \]
\[ = \frac{77597520}{2} = 38,798,760 \]

Pg. 186: 4.22 part a only
Using the standard formulas (page 183 and 184) gives
\[ \mu = \sum x \cdot p(x) = 10(0.05)+20(0.20)+30(0.30)+40(0.25)+50(0.10)+60(0.10) \]
\[ = 0.5+4.0+9.0+10.0+5.0+6.0 = 34.5 \]
\[ \sigma^2 = \sum (x-\mu)^2 \cdot p(x) \]
\[ = (10-34.5)^2 \cdot 0.05 + (20-34.5)^2 \cdot 0.20 + (30-34.5)^2 \cdot 0.30 + (40-34.5)^2 \cdot 0.25 + (50-34.5)^2 \cdot 0.10 + (60-34.5)^2 \cdot 0.10 \]
\[ = (-24.5)^2 \cdot 0.05 + (-14.5)^2 \cdot 0.20 + (-4.5)^2 \cdot 0.30 + (5.5)^2 \cdot 0.25 + (15.5)^2 \cdot 0.10 + (25.5)^2 \cdot 0.10 \]
\[ = 174.75 \]
\[ \sigma = \sqrt{\sigma^2} = \sqrt{174.75} = 13.21930 \]

Pg. 201: 4.52, also a new part f and g
a) If the psychic is guessing (no ESP) what is the value of p, the probability of a correct decision on each trial? Only 1 out of 10 boxes is correct, so 1/10.

b) If the psychic is guessing, what is the expected number of correct decisions in seven trials? This is a binomial experiment with p=0.1 and n=7, so \[ \mu = np = 7(0.1) = 0.7 \] correct decisions.

c) If the psychic is guessing, what is the probability of no correct decisions in seven trials? This is a binomial experiment with p=0.1 and n=7, the problem is asking P(X=0) = \( \binom{n}{x} p^x (1-p)^{n-x} \)
\[ = (7 \text{ choose } 0)(0.1)^0(1-0.1)^7 = 1 (1) (0.9)^7 = 0.4782969 \] You could also answer this one by using just the multiplication rule, and you would end up with the same (0.9)^7.

d) Now suppose the psychic has ESP with p=0.5. What is the probability of no correct decisions in seven trials? Same as in c, but with a new p. \[ P(X=0) = (7 \text{ choose } 0)(0.5)^0(1-0.5)^7 = 1 (1) (0.5)^7 = 0.0078125 \]

e) If the psychic failed on all seven trials, is this evidence against them having ESP? Explain. Yes. The chance of them getting none correct if they really have ESP of any worth is only 7 out of a 1,000. That isn’t very likely!

f) What is the probability that the psychic would get exactly two correct if they had no ESP? This one would be really annoying to do out the long way, but is easy if you use the binomial formula.
\[ P(X=2) = (7 \text{ choose } 2)(0.1)^2(1-0.1)^5 = 7!/(2!5!) (0.1)^2(0.9)^5 = 21 (0.1)^2(0.9)^5 = 0.1240029 \]

g) What is the probability that the psychic would get exactly two correct if they had ESP with p=0.5?
\[ P(X=2) = (7 \text{ choose } 2)(0.5)^2(1-0.5)^5 = 21 (0.5)^2(0.5)^5 = 0.1640625 \]