Example 3.4.

An electronics manufacturer claims that 10% of power supply needs servicing during the warrant period. Techinicians at a testing lab purchase 30 units and simulates the usage during the warranty period. We interpret

- power supply unit = "trial"
- supply unit needs servicing during warranty period. = "success"
- $p = P(\text{"success"}) = P(\text{supply unit needs servicing}) = 0.1$.

If the Bernoulli trial assumption holds, then

Y = the number of units requiring

service during warranty period.

$\sim b(n=30, p=0.1)$

(a) What is the probability that exactly five of the 30 power supply units require servicing during the warranty period?

$$p_Y(5) = P(Y=5)$$

$$= \binom{n}{y} p^y (1-p)^{n-y}$$

$$= \binom{30}{5} (0.1)^5 (0.9)^{25} = \cancel{0.10236}$$

$$= 0.102348$$

$\boxed{\text{Rcode}}$

$p_Y(y) = P(Y=y) =$ dbinom(y, n, p)

$\downarrow$

density (point probability)

$= \text{dbinom}(3, 30, 0.1)$

$F_Y(y) = P(Y \le y) = \text{pbinom}(y, n, p)$

(b) What is the probability at most 5 of the 30 power supply units require service

$F_Y(5) = P(Y \le 5) = P(Y=0) + P(Y=1) + P(Y=2) + \cdots + P(Y=5)$

$$= \sum_{y=0}^{5} P(Y=y)$$

$$= \sum_{y=0}^{5} \binom{30}{y} p^y (1-p)^{30-y}$$

$$= \sum_{y=0}^{5} \binom{30}{y} 0.1^y \, 0.9^{30-y}$$

or $P(Y \le 5) = \text{pbinom}(\underset{y}{5}, \underset{n}{30}, \underset{p}{0.1})$

(b) What is the probability at least 5 of the 30 power supply units require service?

$$P(Y \geq 5) = P(Y = 5) + P(Y = 6) + \cdots + P(Y = 30)$$

$$= 1 - \left[(P(Y=4)) + P(Y=3) + \cdots P(Y=0) \right] = 1 - P(Y \leq 4)$$

$$= 1 - \sum_{y=0}^{4} P_Y(y)$$

$$= 1 - \sum_{y=0}^{4} \binom{30}{y} 0.1^y (0.9)^{30-y}$$

if you use R.

or $= 1 - \text{pbinom}(4, 30, 0.1)$

(d). what is $P(2 \le Y \le 8)$

$P(Y \le 8) \rightarrow$ pbniom

$P(Y \ge 8) = 1 - P(\cancel{\phi}\, Y \le 7) = 1 -$ pbnonim

$P(2 \le Y \le 8)$ You cannot do that.

$$P(2 \le Y \le 8) = \sum_{y=2}^{8} P(Y=2) + \cdots + P(Y=8)$$

$$= \sum_{y=2}^{8} P(Y=y)$$

$$= \sum_{y=2}^{8} \binom{30}{y} (0.1)^y (0.9)^{30-y}$$

To use R

① ~~dbi~~ dbinom(2, 30, .1) + ⋯ + dbinom(8, 30, 0.1)

= 0.8143.

② sum(dbinom(2:8, 30, .1))

The dbinom(2:8, 30, 0.1) command

colon.

creates a vector containing $p_Y(2), \dots p_Y(8)$,

and the sum command adds them up.

(3) $P(2 \le Y \le 8)$

$= \cancel{P(}$

$\underbrace{P(Y \le 8)}_{\text{from } 0 \text{ to } 8} - \underbrace{P(Y \le 1)}_{\text{from } 0 \text{ to } 1}$ = pbinom − pbinom.

from 2 to 8.

3.3 Geometric distribution.

"How many times do you need to try

until you see the sucess."

The geometric distribution also arises in experiment involving Bernoulli trials.

1. each trial Results in a "sucess" or a "failure"

2. The trials are ~~important~~ independent.

3. The probability of sucess, p, where $0 < p \leq 1$, is the same on each trial.

$\boxed{\text{Geometric distribution}}$

Suppose that Bernoulli trials are continuously observed, Define

$Y = \#$ trials to observe the <u>first</u> success.

[if $Y=5$, "ffffs"]

We say that Y has a geometric distribution with success probability p.

$$Y \sim geom(p).$$

pmf: If $Y \sim geom(p)$, the pmf of ~~g~~ Y is given by

$$p_Y(y) = \begin{cases} (1-p)^{y-1} \cdot p & y = 1, 2, \dots \\ 0, & o.w. \end{cases}$$

Mean/Variance

If $Y \sim geom(p)$, then

$$E(Y) = \frac{1}{p}$$

$$Var(Y) = \frac{1-p}{p^2}$$

Example. ~~Bio~~ ~~Bio~~ Biology students are checking the eye color of fruit flies. For each fly, the prob of observing white eyes is $p=0.25$.

We interpret. ~~AA~~ Aa Aa

AA Aa Aa aa

25%

fruit-fly = "trial"

fruit fly has white eyes = "success"

$p = P(\text{success}) = P(\text{white eyes}) = 0.25$

If Bernoulli trial assumptions ~~holds~~ hold,

then Y = # of flies needed to find the first white eyed.

$\sim \text{geom}(0.25)$

(a) What is the prob the first white eyed fly is observed on the 5th fly checked,

$$p_Y(5) = P(Y=5) = (1-0.25)^{5-1}\, 0.25$$
$$= (1-p)^{y-1} p$$
$$\doteq 0.079$$

(b) What is the probability the first whited-eyed fly is observed before the 4th fly is examined?

Note: For this to happen, we must observe the first white-eyed fly (success) on either the 1st, 2nd or 3rd fly.

$$F_Y(3) = P(Y \le 3) = P(Y=1) + P(Y=2) + P(Y=3)$$

$$= (1-0.25)^{1-1}\, 0.25 + (1-0.25)^{2-1}\, 0.25 + (1-0.25)^{3-1}\, 0.25 + \cancel{(1-0.25)^{4-1}\, 0.25}$$

$= 0.25 + 0.1875 + 0.140625 \doteq 0.578$

R Code to do this

CDF

$p_Y(y) = P(Y = y)$

$F_Y(y) = P(Y \le y)$

dgeom(y-1, p)

$=$ pgeom(y-1, p)

↓

failed counts

"fffs"

↓

Y = 4, failed counts = 3.

3.4. Negative Binomial distribution. [Geometrical Generalization]

Note: The negative B distribution also arises from experiments involving Bernoulli trials

1. Binary 2. Independent 3. same p

Negative Binomial Distribution.

Suppose Bernoulli trials are continuously observed. Define.

$Y=$ the number of trials to observe the rth success

We say that Y follows a negative Binomial distribution with waiting parameter r, and success probability p. Notation,

$$Y \sim nb(r, p)$$

<u>Remark.</u>

The negative Binomial distribution is a generalization of the geometric. If $r=1$, then ~~ge~~ $nb(1, p)$ reduces to the geom(p).

[In Binomial, successes are random.

but trials are fixed.

In nB, the trials are ~~fixed~~, random.

in order to see a fixed number of

success]

$\boxed{\text{pmf:}}$

If $Y \sim$ nb(r, p), then the pmf of Y

is given by

$$p_Y(y) = \begin{cases} \binom{y-1}{r-1} p^r (1-p)^{y-r}, & y = r, r+1, r+2, \dots \\ 0, & \text{o.w} \end{cases}$$

If $Y \sim nb(r, p)$,

Mean: $E(Y) = \frac{r}{p}$

variance $Var(Y) = \frac{r(1-p)}{p^2}$.

Example 3.6

At an automotive paint plant, 15% of all batches sent to the lab for chemical analysis don't conform specifications. In this situation, we interpret

- Batch = "trial"
- Batch does not conform = "success"
- P ("success") $= p =$ P("not con") $= 0.15$.
  trials

If Bernoulli assuption hold, then

Y = # of trials needed to

find the 3rd non-conforming Batch

$\sim nb(r=3, p=0.15)$

(a) see (a) below

(b) What is the probability of no more than two non-conforming batches will be observed among the first 30 batches send to the Lab?

↳ This means the third non-conforming batche (success) must be observed on the 31st batche tested, the 32nd, 33rd, ...

$$P(Y \geq 31) = \cancel{P(Y} \; 1 - P(Y \leq 30)$$

$$= 1 - \sum_{y=3}^{30} \binom{y-1}{3-1} (0.15)^3 (0.85)^{y-3}$$

$$= 0.151.$$

However:

If we define Y as successes we observed, not the trials needed,

then $Y \sim b(30, 0.15)$

$= P(Y=0) + P(Y=1) + P(Y=2)$

$= \sum$ dbinom(0:2, 30, 0.15)

$= 0.1514$

R-code for negative Binomial

$P_Y(y) = P(Y=y)$

dnbinom(y-r, r, p)

$F_Y(y) = P(Y \le y)$

$=$ pnbinom(y-r, r, p)

[Solution of (a)

dnbinom(10-3, 3, 0.15)

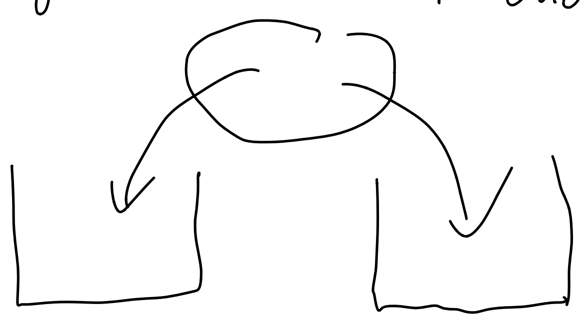
(a) what is the prob the third non-conforming batch is observed at the 10th batch sent to the lab.

$$P_Y(10) = P(Y=10) = \binom{10-1}{3-1}(0.15)^3(1-0.15)^{10-3}$$

$$= \binom{9}{2} \cdot (0.15)^3 (0.85)^7$$

$$\approx 0.039.$$

3.5 Hypergeometric distribution.



Spit one population into 2 jars.

$\boxed{\text{Setting}}$ Consider a population of n objects and suppose that each object belongs to one of two ~~(b)~~ dichotomous classes: Class 1 & Class 2.

E.g. Objects might be people (infected / not)

E.g. objects might be parts (conforming / not)

E.g. plots of land (Respond to treatment / not)

Then in the population of interest, we have

N = Total # of objects

r = number of objects in C_1

$N-r$ = number of objects C_2

Envision: taking a sample of n objects from the population. [objects are selected AT RANDOM and without replacement]

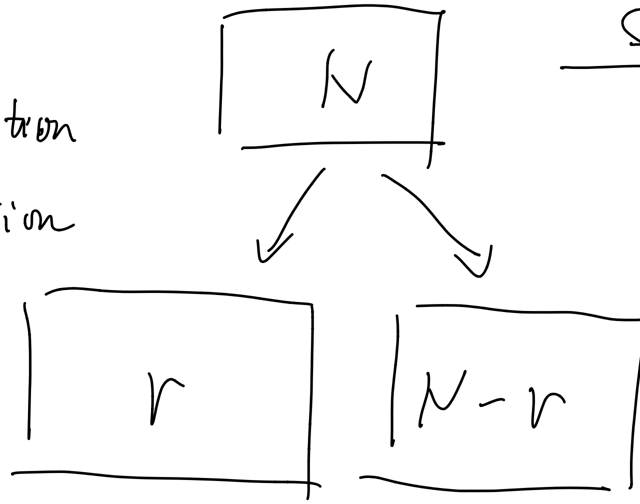
Define:

Y = # of objects in class 1 (out of n selected)

We then say Y has a hypergeometric distribution

$$Y \sim \text{hyper}(N, n, r)$$

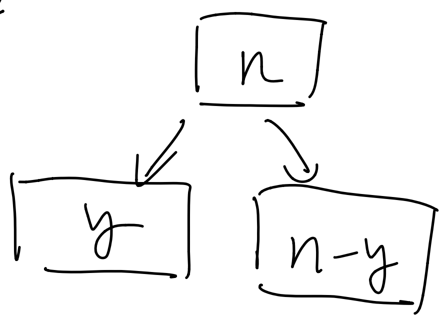
population version



$\xrightarrow{\text{sample}}$ n

contains both class 1 and class 2 object.

sample version



prob of this

use number examples !!!

pmf If $Y \sim hyper(N, n, r)$. then pmf of Y is given by

$$P_Y(y) = \begin{cases} \dfrac{\binom{r}{y}\binom{N-r}{n-y}}{\binom{N}{n}} & y \le r, \ \& \ n-y \le N-r \\ 0, & o.w. \end{cases}$$

~~Mean~~: If $Y \sim hyper(N, n, r)$

Var $n\left(\frac{r}{N}\right)$

Mean : $n\left(\frac{r}{N}\right)\left(\frac{N-r}{N}\right)\left(\frac{N-n}{N-1}\right)$

Example 3.7 .

A