Wednesday, June 29, 2016

# Homework 1

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STAT 512: Mathematical Statistics

Deadline: July, 11TH, Before Class

# Question 1

Let $Y$ be a random variable with probability density function given by

$$f(y) = \begin{cases} 2(1-y), & 0 \leq y \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

**a** Find the density function of $U_1 = 2Y - 1$.

**b** Find the density function of $U_2 = 1 - 2Y$.

**c** Find the density function of $U_3 = Y^2$.

# Question 2

The amount of flour used per day by a bakery is a random variable $Y$ that has an exponential distribution with mean equal to 4 tons. The cost of the flour is proportional to $U = 3Y + 1$.

**a** Find the probability density function for $U$.

**b** Use the answer in part (a) to find $E(U)$.

# Question 3

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The waiting time $Y$ until delivery of a new component for an industrial operation is uniformly distributed over the interval from 1 to 5 days. The cost of this delay is given by $U = 2Y^2 + 3$. Find the probability density function for $U$.

# Question 4

Assume that $Y$ has a beta distribution with parameters $\alpha$ and $\beta$.

**a** Find the density function of $U = 1 - Y$.

**b** Identify the density of $U$ as one of the types we studied in Chapter 4. Be sure to identify any parameter values.

**c** How is $E(U)$ related to $E(Y)$?

# Question 5

Suppose that $Y$ has a gamma distribution with parameters $\alpha$ and $\beta$ and that $c > 0$ is a constant.

**a** Derive the density function of $U = cY$.

**b** Identify the density of $U$ as one of the types we studied in Chapter 4. Be sure to identify any parameter values.

**c** The parameters $\alpha$ and $\beta$ of a gamma-distributed random variable are, respectively, "shape" and "scale" parameters. How do the scale and shape parameters for $U$ compare to those for $Y$?

Note: Use MGF method to find density function of $U$.