Thursday, July 21, 2016

Homework 3

STAT 512: Mathematical Statistics

Deadline: July, 28TH, Before Class

Question 1

A forester studying the effects of fertilization on certain pine forests in the Southeast is interested in estimating the average basal area of pine trees. In studying basal areas of similar trees for many years, he has discovered that these measurements (in square inches) are normally distributed with standard deviation approximately 4 square inches. If the forester samples $n = 9$ trees, find the probability that the sample mean will be within 2 square inches of the population mean.

Question 2

Ammeters produced by a manufacturer are marketed under the specification that the standard deviation of gauge readings is no larger than .2 amp. One of these ammeters was used to make ten independent readings on a test circuit with constant current. If the sample variance of these ten measurements is .065 and it is reasonable to assume that the readings are normally distributed, do the results suggest that the ammeter used does not meet the marketing specifications? [*Hint:* Find the approximate probability that the sample variance will exceed .065 if the true population variance is .04.]

Question 3

Suppose that independent samples (of sizes n_i) are taken from each of k populations and that population i is normally distributed with mean μ_i and variance σ^2, $i = 1, 2, \ldots, k$. That is, all populations are normally distributed with the *same* variance but with (possibly) different means. Let $\overline{X}_i$ and S_i^2, $i = 1, 2, \ldots, k$ be the respective sample means and variances. Let $\theta = c_1\mu_1 + c_2\mu_2 + \cdots + c_k\mu_k$, where $c_1, c_2, \ldots, c_k$ are given constants.

a Give the distribution of $\hat{\theta} = c_1\overline{X}_1 + c_2\overline{X}_2 + \cdots + c_k\overline{X}_k$. Provide reasons for any claims that you make.

b Give the distribution of

$$\frac{\text{SSE}}{\sigma^2}, \quad \text{where SSE} = \sum_{i=1}^{k}(n_i - 1)S_i^2.$$

Provide reasons for any claims that you make.

Question 4

Let $Y_1, Y_2, \ldots, Y_5$ be a random sample of size 5 from a normal population with mean 0 and variance 1 and let $\overline{Y} = (1/5)\sum_{i=1}^{5} Y_i$. Let Y_6 be another independent observation from the same population. What is the distribution of

a $W = \sum_{i=1}^{5} Y_i^2$? Why?

b $U = \sum_{i=1}^{5} (Y_i - \overline{Y})^2$? Why?

c $\sum_{i=1}^{5} (Y_i - \overline{Y})^2 + Y_6^2$? Why?

Question 5

If Y has an exponential distribution with mean θ, show that $U = 2Y/\theta$ has a χ^2 distribution with 2 df.

Question 6

A plant supervisor is interested in budgeting weekly repair costs for a certain type of machine. Records over the past years indicate that these repair costs have an exponential distribution with mean 20 for each machine studied. Let $Y_1,\ Y_2, \dots, Y_5$ denote the repair costs for five of these machines for the next week. Find a number c such that $P\left(\sum_{i=1}^{5} Y_i > c\right) = .05$, assuming that the machines operate independently.

Hint: Use the result from Question 5, and find c from a chi-table.