Assumptions and Relation between Confidence Interval and Hypothesis Test
On t based Inference for \( \mu \)

Assumptions for Validity of Confidence Interval and Hypothesis test for \( \mu \)

- Data must be from a random sample from large population
- Observations in the sample must be independent of each other
- \( n \) small, population distribution must be approximately normal
- \( n \) large, population need not be approximately normal (CLT kicks in)

A statistical procedure is said to be robust if the results of the procedure are not affected very much when the conditions for validity are violated.

The \( t \) procedures are fairly robust to non normality except in the case of outliers or strong skewness. Why?

The following are some loose guidelines:
**Relationship between Confidence Interval and Hypothesis Test**

Draw two pictures: The hypothesis test corresponding to $H_A: \mu \neq \mu_0$ when we

- Reject $H_0$
- Fail to reject $H_0$

When we fail to reject, we have the following inequality:

And this should look familiar...

So, the events that lead to the decision to fail to reject $H_0$ for the two-sided test are exactly the events that form the $(1-\alpha)\%$ confidence interval for $\mu$.

**The Moral** If the confidence interval contains $\mu_0$, then we would fail to reject $H_0$ for the two-sided test of $H_0: \mu = \mu_0$ against $H_A: \mu \neq \mu_0$ and *vice versa*. 