Binomial Distribution

Let’s refresh our memory on computing probability with an example. Consider the experiment where we toss a fair coin 3 times. Find the probability distribution for flipping “heads” in this experiment.

Now, think about finding probability distributions associated with flipping a fair coin say 6 times. And then consider the experiment where the coin is not fair. The calculations get unwieldy fast! We need a more convenient method...
**Definition:** The **independent trials model** occurs when

(i) n independent trials are studied
(ii) each trial results in a single binary observation
(iii) each trial’s success has (constant) probability: \( P\{\text{success}\} = p \)

Notice that if \( P\{\text{success}\} = p \), \( P\{\text{failure}\} = 1 - p \).

Your text calls this the **BInS (Binary / Indep. / n is constant / Same p)** setting, but is commonly referred to as a **Binomial Experiment**

In a BInS setting, if we let \( Y = \{\# \text{ successes}\} \) then \( Y \) has a **binomial distribution**.

**NOTATION:** \( Y \sim \text{Bin}(n,p) \).

The binomial probability function is

\[
P\{Y = j\} = \binom{n}{j} p^j (1-p)^{n-j} \quad j = 0,1,...,n
\]

where \( \binom{n}{j} = \frac{n!}{j!(n-j)!} \) with \( j! = j(j-1)(j-2)\ldots(2)(1) \) and define \( 0! = 1 \)

**Example** Use the binomial probability function to find \( P\{\text{exactly 1 head}\} \) in the experiment where a fair coin is flipped 3 times.

Find \( P\{\text{at least one head}\} \)
The TI calculators will compute binomial probabilities.

For \( P(Y = j) \) Choose 2\(^{nd}\) and VARS to bring up “DISTR” menu -> scroll down to binompdf -> ENTER -> binompdf(n,p,j)

For \( P(Y \leq j) \) choose 2\(^{nd}\) and VARS to bring up “DISTR” menu -> scroll down to binomcdf -> ENTER -> binomcdf(n,p,j)

Example 3.43 Suppose that 39% of the individuals in a large population have a certain mutant trait and that a random sample of 5 individuals is chosen from the population.

Find \( P(\text{at least 1 and at most 4 mutants}) \) in the sample

The following is a probability histogram of the distribution from example 3.43.

![Probability Histogram](image)

**Figure 3.15** Binomial distribution with \( n = 5 \) and \( p = .39 \)

**Binomial mean and variance**

If \( Y \sim \text{Bin}(n,p) \), the population mean and variance are:

\[
\mu_Y = np \quad \text{and} \quad \sigma_Y^2 = np(1-p)
\]

Example Find the mean and standard deviation for the number of mutants out of five selected individuals from the population described in example 3.43