

$$\tilde{p} \pm Z_{\alpha/2} \sqrt{\frac{\tilde{p}(1-\tilde{p})}{n + Z_{\alpha/2}^2}} \quad \text{where } \tilde{p} = \frac{Y + \frac{1}{2}Z_{\alpha/2}^2}{n + Z_{\alpha/2}^2}$$

$$(\tilde{p}_1 - \tilde{p}_2) \pm Z_{\alpha/2} \sqrt{\frac{\tilde{p}_1(1-\tilde{p}_1)}{n_1 + 2} + \frac{\tilde{p}_2(1-\tilde{p}_2)}{n_2 + 2}} \quad \text{where } \tilde{p}_1 - \tilde{p}_2 = \frac{Y_1 + 1}{n_1 + 2} - \frac{Y_2 + 1}{n_2 + 2}$$

$$\sum_{i=1}^n \frac{(O_i - E_i)^2}{E_i}$$

$$\frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2}$$

$$\bar{y} - b_1 \bar{x}$$

$$\sqrt{\frac{SS(resid)}{n-2}}$$

$$\frac{S_{Y|X}}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2}}$$

$$\frac{[\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})]^2}{\sum_{i=1}^n (x_i - \bar{x})^2 \sum_{i=1}^n (y_i - \bar{y})^2}$$

This exam is worth a total of 120 points.

Part I: Answer eight of the following nine questions. If you complete more than eight, I will grade only the first eight. Five points each.

1) State the definition of a P-value.

The probability under  $H_0$  of observing a test statistic as extreme or more extreme (in the direction of  $H_A$ ) as that actually observed.

2) The Central Limit Theorem says that for any i.i.d. random sample,  $Y_1, Y_2, \dots, Y_n$  where  $E[Y_i] = \mu$  and  $E[(Y_i - \mu)^2] = \sigma^2$ , then as  $n \rightarrow \infty$  the distribution of the sample mean is **normal** with mean,  $\underline{\mu}$ , and variance,  $\frac{\sigma^2}{n}$  (note, I'm asking for variance here – not standard deviation).

3) What are the degrees of freedom for a Chi-square test of association for two categorical variables using a 4 x 4 contingency table?

$$df = (4-1)(4-1) = 9$$

4) (Circle the correct answer.) When collecting four independent random samples, each from different regions of the country, and recording whether the individuals have immunity to the Epstein-Barr virus or not, which of the following is the correct  $H_0$  to choose for the Chi-square test using the 2 x 4 contingency table?

$H_0$ : prevalence of Epstein-Barr immunity is the same for all four regions

$H_0$ : region and Epstein-Barr immunity are not associated

5) Recall the snapdragon example from the class slides used to illustrate the goodness of fit test. In that example, snapdragon progeny of pink (hybrid) parents is expected to appear in a 1 red: 2 pink : 1 white ratio. We tested  $H_0$ : the model fits against  $H_A$ : the model fit is poor.

Is it valid to test  $H_0$ : the model fits against  $H_A$ : model fit is poor with proportions of red and white being lower than predicted and proportion of pink being higher than predicted? Why or why not?

No, it would not be valid. The hypothesis that  $\Pr\{\text{red}\}=0.25$ ,  $\Pr\{\text{pink}\}=0.5$ ,  $\Pr\{\text{white}\}=0.25$  is an example of a compound hypothesis (two independent assertions are being made – see text, page 396). Directional alternatives may only be considered for the Chi-square Goodness-of-fit test when we have only two categories (and the snapdragon example has three).

6) (Circle the correct answer.) Suppose we have a data set of  $(x_i, y_i)$  pairs and X and Y are both random variables. Assuming assumptions are met, it is valid to conduct

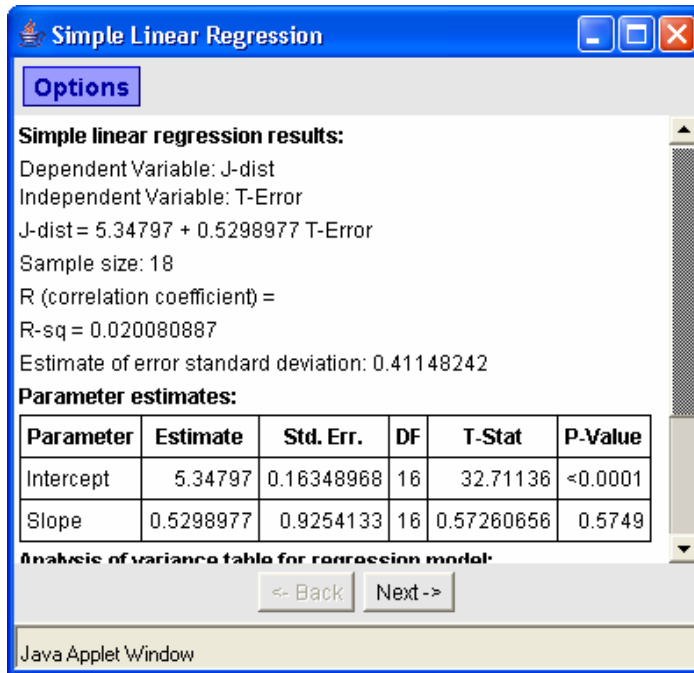
**simple linear regression analysis only**

**correlation analysis only**

**either of simple linear regression or correlation**

**Questions 7,8, and 9 are on the next page!**

The long jump is a track and field event in which the competitor attempts to jump a maximum distance into a sandpit after a running start. At the edge of the sandpit is a takeoff board. Jumpers usually plant their toes at the front edge of this board to maximize jumping distance. The distance between the front edge of the board and where the toe actually lands on the board prior to the jump is called “takeoff error”. Kinesiology researchers videotaped 18 novice long jumpers and measured takeoff error and jump distance for these individuals. Use DoStat’s simple linear regression output (below) to answer questions 7, 8, and 9.



7) What is the correlation coefficient (r) for this regression?

$r = \pm\sqrt{r^2} = \sqrt{0.020080887} = 0.1417$  (and it's positive since  $b_1$  is positive)

8) State and interpret the value of the coefficient of determination ( $r^2$ ) in the context of the setting.

2% of the variation in jump distance is explained by the variation in takeoff error.

9) Report the P-value for the test of

$H_0: \rho = 0$

$H_A: \rho \neq 0$

P-value: 0.5749 (test of  $H_0: \rho = 0$  is numerically equivalent to test of  $H_0: \beta_1=0$ )

Part II: Answer every part of the next three problems. Read each question carefully, and show your work for full credit.

1) The typical procedure of treating moderately advanced cancer of the larynx is surgical removal. Attempts have been made to treat cancer of the larynx by radiation therapy alone, thereby saving the patient's voice box. Data on a group of patients treated by this method at the University of Florida's Shands Hospital were reported in the *International Journal of Radiation Oncology and Biological Physics* (Vol. 10, 1984). Of the 18 patients treated with radiation only, 15 cancers were ultimately controlled at the site (the larynx). Ultimate control of the cancer was achieved for 21 of the 23 patients who were treated by surgery alone.

1a) (15 points) Construct a 95% Agresti-Caffo confidence interval for the difference in the proportion of controlled cancer at the larynx for these two groups.

$$\tilde{p}_1 = \frac{15+1}{18+2} \quad \tilde{p}_2 = \frac{21+1}{23+2}$$

$$(\tilde{p}_1 - \tilde{p}_2) \pm Z_{\alpha/2} \sqrt{\frac{\tilde{p}_1(1-\tilde{p}_1)}{n_1+2} + \frac{\tilde{p}_2(1-\tilde{p}_2)}{n_2+2}}$$

$$\left(\frac{16}{20} - \frac{22}{25}\right) \pm 1.96 \sqrt{\frac{16/20 \cdot 4/20}{20} + \frac{22/25 \cdot 3/25}{25}}$$

$$\mathbf{-0.08 \pm 1.96(0.1105622)}$$

**Our 95% confidence interval is (-0.297, 0.137)**

1b) (10 points) Interpret the interval you just computed in part (a).

With 95% confidence, we are unsure whether the proportion of patients whose cancer of the larynx is ultimately cured is different for either treatment.

If the proportion is bigger under the radiation only treatment, it is by as much as 0.137.

If the proportion is bigger under the surgery only treatment, it is by as much as 0.297.

2) (20 points) Researchers interested in whether mental exercise could build “mental muscle” conducted an experiment using 12 littermate pairs of young male rats. One member of each pair, chosen at random, was raised in an “enriched” environment with toys and companions, while its littermate was raised alone in an “impoverished” environment. After 80 days, the rats were sacrificed and their brains were dissected by a researcher who did not know which treatment each rat had received. The weight of the cerebral cortex, expressed relative to total brain weight, was measured. For 10 of the 12 pairs, relative weight of the cerebral cortex was bigger for the rat from the “enriched” environment than for his “impoverished” littermate. In the other 2 pairs, the “impoverished” rat had the larger cortex. The results are summarized in the table below.

“enriched” bigger	“impoverished” bigger
10 (6)	2 (6)

Use a goodness-of-fit test with  $\alpha = 0.05$  to test the hypothesis that cerebral cortex size is the same for rats in either environment against the alternative that cerebral cortex size is larger for rats with an “enriched” environment.

(1)  $\alpha = 0.05$

(2)  $H_0$ : cerebral cortex size is the same for rats in either environment (ratio is 1:1)  
 $H_A$ : cerebral cortex size is bigger for rats in the “enriched” environment

(3) Under the null hypothesis (that nothing is going on), we’d expect for the cerebral cortex to be bigger about 50% of the time for the rats in the “enriched” environment (1/2 of 12 = 6) and bigger the other 50% of the time for the rats in the “impoverished” environment (1/2 of 12 = 6). Using these expected counts, we derive out test statistic:

$$X^2 = \frac{(10 - 6)^2}{6} + \frac{(2 - 6)^2}{6} = 5.3333$$

(4)  $P = \frac{1}{2}(\Pr\{\chi^2_{df=1} \geq 5.3333\}) = \frac{1}{2}(0.0209) = 0.01045$  (DoStat calculator)  
 Or  $0.02 < 2P < 0.050 \Rightarrow 0.01 < P < 0.025$  (Table 9)

(5)  $0.01045 = P < \alpha = 0.05$ , reject  $H_0$

(6) There is significant evidence at the 0.05 significance level that cerebral cortex size is larger for rats who live in the “enriched” environment.

3) An exercise physiologist used skinfold measurements to estimate the total body fat expressed as a percentage of body weight, for 9 female participants in a physical fitness program. The body fat percentages and the body weights are shown in the table.

Participant	Weight X (kg)	Fat Y (%)
1	57	29
2	68	32
3	69	35
4	59	31
5	62	29
6	59	26
7	56	28
8	66	33
9	72	33

Preliminary calculations yield the following results:

$$\bar{x} = 63.111$$

$$\bar{y} = 30.667$$

$$\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y}) = 108.3334$$

$$\sum_{i=1}^n (x_i - \bar{x})^2 = 268.889$$

$$SS(\text{resid}) = 22.3533$$

3a) (7 points) Calculate the least-squares regression line using X = weight as the predictor variable and Y = body fat as the response.

$$b_1 = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2} = \frac{108.3334}{268.889} = 0.40289$$

$$b_0 = \bar{y} - b_1 \bar{x} = 30.667 - (0.40289)(63.111) = 5.240$$

Our line is  $Y = 5.240 + 0.403X$

3b) (7 points) Calculate the residual standard deviation ( $S_{Y|X}$ ).

$$\sqrt{\frac{SS(\text{resid})}{n-2}} = 1.787\%$$

3c) (7 points) Give an estimate of the mean and standard deviation of female percent body fat at a weight of 63 kg.

$$\hat{\mu}_{Y|X} = 5.240 + 0.403(63) = 30.629\%$$

$$S_{Y|X} = 1.787\%$$

3d) (7 points) Calculate a 95% confidence interval for  $\beta_1$  (slope of the true line).

$$b_1 \pm t_{\alpha/2} \frac{S_{Y|X}}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2}}$$

$$0.40289 \pm 2.365(0.1089778)$$

$$(0.145, 0.661)$$

3e) (7 points) Interpret the interval you just computed in part (d).

We are 95% confident that for every one kilogram increase in weight, we expect the mean body fat percent to increase as little as 0.145 or as much as 0.661 for women in a fitness program like the one in this study.