STAT 205
Spring 2007
Exam 2

Name: ______ANSWER KEY_

$$P\{Y = j\} = {}_nC_j\, p^j\, (1 - p)^{n-j}$$

$$Z = \frac{\bar{Y} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$\bar{Y} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

$$t = \frac{\bar{Y} - \mu}{\frac{s}{\sqrt{n}}}$$

$$(\bar{Y}_1 - \bar{Y}_2) \pm t_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$t = \frac{(\bar{Y}_1 - \bar{Y}_2) - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Part I: Answer eight of the following nine questions. If you complete more than eight, I will grade only the first eight. Five points each.

1) State the definition of a P-value.
The probability under H_0 of observing a test statistic as extreme or more extreme (in the direction of H_A) as that actually observed.

2) (Circle the correct answer) A hypothesis test has been conducted at the 0.05 significance level, resulting in a P-value of 0.06. Obviously, in this case we fail toreject H_0. If an error was made, it would be a **Type I** / **Type II** / **neither** error.

3) Suppose you want to analyze a sample of size n=10 and your QQplot shows a violation of normality. In practice, a transformation is commonly attempted before using a non-parametric test.

4) (Circle one) True / False A *t*-distribution based confidence interval for the mean, μ, and a *t*-distribution based hypothesis test for the mean, μ, are different forms of the same inference.

5) (Circle the correct answer) Twenty institutionalized epileptic patients participated in a study of a new anticonvulsant drug Valproate. Ten of the patients (chosen at random) were started on the daily Valproate and the other ten received an identical placebo pill. During an eight week observation period, the number of major and minor seizures was counted. After this, the patients were "crossed over" to the other treatment, and seizure counts were made during a second eight week observation period. We would use the **independent** / **dependent (paired)** samples method in order to conduct a test of hypothesis.

6) We learned several standard error of means (SEMs) for use in making inference on the population mean. In the independent samples setting, we have been applying one type of SE for the difference between two means, but you also learned of another type of SE which can be used in the independent samples setting called SE_{pool}. Describe the special situation for which we can deploy its use.
We use SE_{pool} when we have reason to believe the standard deviation for the population group one came from is the same as that for the population of group two ($\sigma_1 = \sigma_2$).

7) State the assumptions required for validity of a *t* confidence interval on the population mean, μ.

- Data collected from a random sample from a large population
- Observations in the sample must be independent from each other
- n small, population distribution must be approximately normal
- n large, population need not be approximately normal (CLT kicks in)

8) A geneticist weighed 28 female lambs at birth. The lambs were all born in April, were all the same breed (Rambouillet), and were all single births (no twins). The diet and other environmental conditions were the same for all the parents. The geneticist reported a *t* distribution based confidence interval for the mean birth weight. If twin births had been included, would the t-based interval still be valid? Why or why not?
No, the confidence interval would not be valid since observations on a set of twins would be dependent (since knowing one's weight would give insight into the other twin's weight). This would violate the assumption that observations within a sample must be independent from each other.

9) The Central Limit Theorem says that for any i.i.d. random sample, $Y_1, Y_2, \ldots, Y_n$ where $E[Y_i] = \mu$ and $E[(Y_i-\mu)^2] = \sigma^2$, then as $n \to \infty$ the distribution of the sample mean is approximately normal with mean, μ, and variance, σ^2/n.

Part II: Answer every part of the next two problems. Read each question carefully, and show your work for full credit.

1) A classical method of researching the nature vs. nurture question is using identical twins to conduct research. A book by Susan Farber (*Identical Twins Reared Apart*, New York: Basic Books, 1981) contains a chronicle and reanalysis of identical twins reared apart. One question of interest is whether there are significant differences in the IQ scores of identical twins where one member of the twin pair is reared by natural parents and the other is not.

1a) (25 points) A random sample of 32 sets of twins under the above circumstances was taken. The sample mean difference (IQ twin reared by natural parent – IQ twin not reared by natural parent) in IQ scores is -2.906, the standard deviation of the differences is 8.895. Conduct a test of hypothesis to see whether there is a significant difference (at the $\alpha = 0.05$ significance level) in the mean IQ scores under these two conditions.

Note: This is the dependent (paired) samples setting.
Let R / NR denoted reared / not reared by natural parent, respectively.

(1) $\alpha = 0.05$

(2) H_0: $\mu_R - \mu_{NR} = 0$ or $\mu_R = \mu_{NR}$ or $\mu_d = 0$
H_A: $\mu_R - \mu_{NR} \neq 0$ or $\mu_R \neq \mu_{NR}$ or $\mu_d \neq 0$

STAT -> scroll over to TESTS -> scroll down to T-Test
Choose "Stats", μ_o:0, $\bar{x}$: -2.906, s_x: 8.895, n: 32, μ: $\neq \mu_o$ -> Calculate -> ENTER

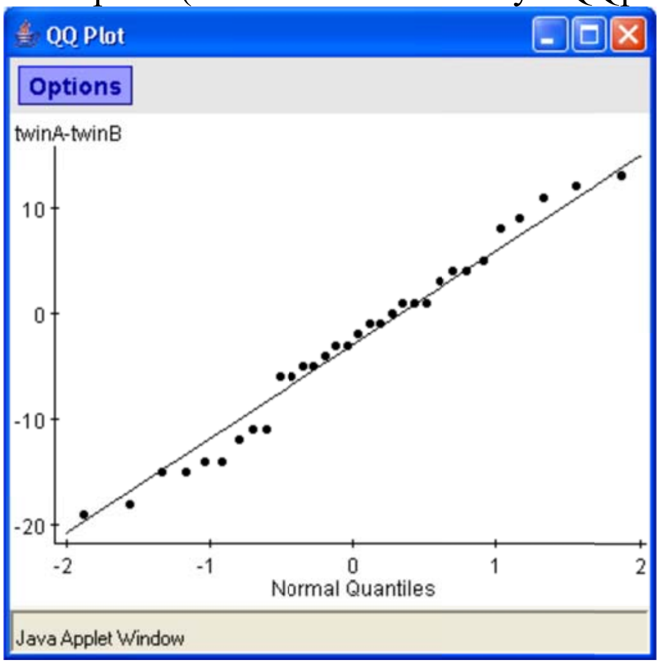
(3) $t_s = -1.848$

(4) P = 0.0741

(5) P > α, so fail to reject H_0

(6) There is not significant evidence (at the $\alpha = 0.05$ significance level) to conclude that when twins are reared separately, true mean IQ score differs for twins reared by natural parents versus twins who are not reared by their natural parents.

1b) (5 pts) Using the QQplot of the differences below, comment on whether the assumption (that can be checked by a QQplot) appears to be met for the *t* procedure.



The points follow the line very closely and do not show a systematic departure from the line (are not making a shape). Even though the points at the high end if the distribution go above the line, they come back (and similarly in the low end of the distribution). Considering this and the fact that we have 32 observations, the CLT should kick in for us (which would allow us to assume the sample mean of the observations is normally distributed).
Conclusion: There is little to no evidence of a violation of the normality assumption.

2) Researchers were interested in the short term effect of caffeine on heart rate. They took a random sample of individuals and measured each person's resting heart rate. Then they had each subject each subject drink a 6 ounce cup of coffee. Nine of the subjects were given coffee containing caffeine and eleven were given decaffeinated coffee. After ten minutes, each person's heart rate was measured again. The data in the table show the change in heart rate (beats per minute) for each subject; a positive number means the heart rate went down and a negative number means the rate went up.

2a) (25 points) Use these data to construct a 90% confidence interval for the difference in mean heart rate change between the caffeinated and decaffeinated conditions. You may proceed as though the assumptions have been checked and deemed acceptable.

Note: This is an independent samples setting since the patients were randomly allocated into one group or the other, caffeinated or decaffeinated (there was only one difference in heart rate noted on each participant).

	Caffeine	**Decaf**
	28	26
	11	1
	-3	0
	14	-4
	-2	-4
	-4	14
	18	16
	2	8
	2	0
		18
		-10
n	**9**	**11**
$\bar{x}$	**7.3**	**5.9**
s	**11.1**	**11.2**

STAT -> TESTS -> 2SampTInt -> if you choose "Stats" enter the x-bar, s, and n for each sample, always keep Freq: 1, , change C-Level: 0.9, choose POOLED: NO and get ($-7.306 < \mu_C - \mu_D < 10.106$)
Or Choose "Data" if you entered your data in List 1 and 2 to get ($-7.278 < \mu_C - \mu_D < 10.127$)

2b) (5 pts) Interpret the interval you computed in part (a).

With 90% confidence, we are unsure there is a difference when comparing true mean change in heart rate within ten minutes after drinking caffeinated coffee or decaf coffee.. If it is larger under the caffeinated condition, it is by as much as 10.127 beats per minute. If it is larger under the decaf condition, it is by as much as 7.278 beats per minute.