Rank-based estimation for GARCH models

Beth Andrews
Department of Statistics, Northwestern University
E-Mail: TBP

Abstract: Observed time series processes frequently appear uncorrelated, yet exhibit volatility clustering. Volatility clustering is the tendency of observations relatively small in absolute value to be followed by other small observations, and the tendency of observations relatively large in absolute value to be followed by other large observations. Hence, these series appear uncorrelated, but dependent. Nonlinear generalized autoregressive conditionally heteroskedastic (GARCH) models, which have time-dependent conditional variances, are often used to describe time series with these features. Processes exhibiting GARCH-type behavior have appeared, for example, in financial time series such as inflation rates and stock prices and, outside of finance, in speech signals, daily and monthly mean temperatures, wind speeds, and atmospheric carbon dioxide concentrations. In this paper, we use a rank-based technique to estimate GARCH model parameters. The rank (R) estimators we consider minimize the sum of mean-corrected model residuals weighted by a function of residual rank; they are similar to the R-estimators proposed by L.A. Jaeckel [Estimating regression coefficients by minimizing the dispersion of the residuals, Ann. Math. Statist. 43 (1972) 1449-1458] for estimating linear regression parameters. R-estimators are, in general, robust and relatively efficient. We show this is true in the case of GARCH parameter estimation. The estimation technique is robust because the R-estimators of GARCH model parameters are $n^{1/2}$-consistent (n represents sample size) and asymptotically normal under very mild conditions. Since the weight function can be chosen so that R-estimation has the same asymptotic efficiency as maximum likelihood estimation, the R-estimators are also relatively efficient. In addition, R-estimation dominates classical Gaussian quasi-maximum likelihood estimation with respect to both robustness and asymptotic efficiency. Simulation results for R-estimation show that the asymptotic theory is indicative of finite, large sample behavior.