Nonparametric Density Estimation for Flowgraph Models With Applications to System Reliability

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Abstract: Statistical flowgraphs (Huzurbazar 2005) provide a methodology for analysis of multistate stochastic models with arbitrary transition time distributions. Flowgraph models are solved using integral transforms (e.g., Laplace transforms) of the transition time densities between adjacent states, which are then combined algebraically to derive transforms for transitions between arbitrary pairs of states; e.g., to derive the transform for the time of passage between two states considered "initial" and "final". This latter transform is then inverted to recover the density or distribution function.

As an example, the process of interest may be the operation of a complex system with repairable components. Its states represent various partial failures, recurrent transitions represent fail/repair cycles, and a final state represents complete system failure. The flowgraph methodology allows data on transitions between adjacent states to be used for inference about total system performance and reliability.

Most prior work on flowgraphs has been parametric: Distribution families (gamma, inverse Gaussian, etc.) for transitions are selected based on prior information and inspection of sample histograms, and parameter values are estimated by maximum likelihood; transforms can then be determined in closed form and inverted either analytically or numerically.

This talk presents nonparametric approaches to flowgraphs using empirical transforms. For example, where the Laplace transform of the density for a transition time T is defined as $L(s) = E[\exp(-sT)] = \int_0^\infty \exp(-st)dF(t)$ (F being the distribution function of T), the empirical transform based on a sample $t_1, \ldots, t_n$ is $\tilde{L}(s) = \frac{1}{n}\sum_{i=1}^{n} \exp(-st_i)$. Empirical transforms can be combined algebraically just as in the parametric case; inversion can only be done numerically, using saddlepoint approximation techniques starting from

the (empirical) moment generating function, or by Fourier inversion of the (empirical) Laplace transform.

After reviewing the theory of empirical transforms, we present three examples of nonparametric flowgraphs in a reliability context:

- A three-state repairable redundant system, involving failure and repair of identical units in parallel.
- A model in which a component may fail either randomly with constant hazard rate, or due to wearout centered around a known point in time.
- The repairable redundant system model with incomplete (censored) data on transition times, which requires imputing transition times for the censored observations.

Approximations derived from these models using empirical transforms are compared with the exact densities. We discuss computational issues, factors affecting accuracy of the results relative to parametric methods, and conclude with a discussion of how nonparametric flowgraph methods may be used either alone or in conjunction with parametric methods as a form of cross-validation.

Reference

Huzurbazar, Aparna V. *Flowgraph Models for Multistate Time-to-Event Data.* Hoboken, N. J.: Wiley-Interscience, 2005.