# Ranks and quantiles in regression and autoregressive models

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*Abstract:* Ranks and quantiles are important tools in the nonparametric statistical inference. Ranks are very convenient for testing, while the quantiles/order statistics are appealing for estimation. Natural is the idea to extend these entities to more general models, preserving their advantages to the largest possible extent. The notion of sample quantiles was first extended to linear models by Koenker and Bassett [8] under the name *regression quantiles* (RQ). They formulated the RQ as a solution of a special parametric linear programming problem, and used the dual linear program as a computational device. Gutenbrunner and Jurečková [1] found that the optimal dual solutions share some properties with the vector of ranks, thus called them the *regression rank scores* (RRS). The main advantages of RQ and RRS are the equivariance of the former and the invariance of the latter, what is not true for the residuals from an estimator and their ranks. This naturally led to the $L$-estimation in the linear regression model based on the RQ's and to the extension of the rank tests in models with the nuisance linear regression based on the RRS. Both RQ and RRS were extended to the linear autoregressive model by Koul and Saleh [9].

The regression quantiles were extended to some nonlinear regression models by Jurečková and Procházka [5]. The RRQ now form a basic tool in econometrics, where they became a part of a general consciousness and are independently developing in various directions, suitable for econometric problems. *Quantile regression* is an often subject of econometric conferences and workshops.

We shall describe some applications of the RRS in testing problems with nuisance regression (including nonlinear) and autoregression, without estimating the nuisance parameters. Surprisingly, the tests work well numerically even under distributions for which we do not yet have a rigorous proof. The studentizing scale statistics based on the RRS enjoy a desirable invariance with respect to the regression (see [7]). The close relation of the ranks and quantiles extends to a multiple setup: The two-step regression quantile,

estimating the slope components separately with the aid of a suitable rank-estimate and then ordering the residuals to get the intercept component, is surprisingly close to the ordinary regression quantile, while it well illustrates its structure. It is even more appealing in the case of the *extreme regression quantile*, where the extreme two-step and the ordinary extreme regression quantiles coincide. For the Gumbel domain of attraction and some other distribution tails, the asymptotic distribution of the intercept component of the extreme regression quantile depends on the design matrix only through the diagonal elements of the hat matrix.

There are still many appealing open questions, and we believe that these concepts will still find many interesting applications.

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